International Journal of Modern Physics B, Vol. 16, No. 23 (2002) 3419–3428

MULTIBAND SUPERCONDUCTIVITY

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Received 10 June 2002

Multi-band superconductivity is investigated by using two-particle Green's function techniques, and equations for coupled states are derived in the framework of a two-band model. These results suggest that superconductivity appears, even if electron–electron interaction is positive. We also present a cooperative mechanism for multi-band superconductivity.

PACS number: 74.20-z

1. Introduction

The theory of superconductivity based on electron–phonon interaction by Bardeen, Cooper and Schrieffer (BCS)[1] has long been well established and it is now the standard theory for superconductivity.[1–4] Recently, non-BCS mechanisms via spin fluctuation, charge fluctuation (plasmon), and electron excitation (exciton) have attracted much interest, in particular, in relation to the possibility of high-T_c superconductivity. These mechanisms have a common characteristic — electron–electron (e–e) interaction is the origin of superconductivity.

After the discovery of the high-T_c copper oxides,[5] Anderson[6] has indeed emphasized the important role played by the e–e interaction. In the past decade many non-BCS theories[7–14] have been proposed, but they do not converge as unified and well-accepted theory yet. On the other hand, a lot of experimental studies on the copper oxides have revealed several characteristics: (1) These species are antiferromagnets before doping, in accord with the importance of e–e interaction, (2) high-T_c superconductivity appears in the intermediate region of metal-insulator transition, and it disappears in the metallic or overdoped region.[15–17] Accumulated experimental results on the superconductor species and other related materials suggest a guiding principle that doping in magnetic systems, more generally charge-transfer

(CT) insulators, may provide several exotic phases, which are (a) ferromagnetic metal or insulator, (b) spin glass, (c) paramagnetic metals, (d) antiferromagnetic metals, (e) ferrimagnetic metal or insulator and (f) charge- or spin-mediated superconductor. Relative stabilities of these phases should be dependent on several factors. This in turn indicates that a systematic theoretical description of such phases and phase transitions is quite hard.

Recently, the importance of many-band effects in high-T_c superconductivity has been pointed out.[18−32] Within the framework of two-particle Green function,[18] it is shown that in the electron–phonon system there exists a new class of so-called coupled states. Numerical calculations have shown that superconducting (SC) gap depends on the number of bands crossing the Fermi level and on the momenta $k_1 + k_2 \neq 0$ of the interacting electrons. Moreover, the temperature dependence of the SC gap for high-T_c superconductors is more complicated than that predicted in the BSC approach. We have also investigated anomalous phases in a two-band model using Green function techniques.[27−29] The expressions of the transition temperature for several phases have been derived, and the approach has been applied to superconductivity in molecular crystals by charge injection as well as to field-induced superconductivity.

In this paper, we investigate superconductivity using two-band model and the two-particle Green function techniques. In the framework of two-band model, the coupled states in the electronic system are investigated, and the conditions when the coupled states can appear are derived. We apply the model to an electron–phonon mechanism for the traditional BSC method, an e–e interaction mechanism for high-T_c superconductivity, and a cooperative mechanism in relation to multi-band superconductivity.

2. Theoretical Background

In this section, we briefly summarize a two-band model for superconductivity and introduce two-particle Green's function. The spectral properties of the model are investigated using the two-particle Green function.

2.1. *Hamiltonian*

We start from the Hamiltonian for two-bands i and j

$$H = H_0 + H_{\text{int}}\,, \tag{1}$$

where

$$H_0 = \sum_{k,\sigma}[(\varepsilon_i - \mu)a^+_{i\mathbf{k}\sigma}a_{i\mathbf{k}\sigma} + (\varepsilon_j - \mu)a^+_{j\mathbf{k}\sigma}a_{j\mathbf{k}\sigma}]\,, \tag{2}$$

$$H_{\text{int}} = \frac{1}{4}\sum_{\delta(\mathbf{p}_1+\mathbf{p}_2,\mathbf{p}_3+\mathbf{p}_4)}\sum_{\alpha\beta\gamma\delta}[\Gamma^{iiii}_{\alpha\beta\gamma\delta}a^+_{i\mathbf{p}_1\alpha}a^+_{i\mathbf{p}_2\beta}a_{i\mathbf{p}_3\gamma}a_{i\mathbf{p}_4\delta} + (i \to j)$$

$$+\Gamma^{iijj}_{\alpha\beta\gamma\delta}a^{+}_{i\mathbf{p}_1\alpha}a^{+}_{i\mathbf{p}_2\beta}a_{j\mathbf{p}_3\gamma}a_{j\mathbf{p}_4\delta}+(i\to j)$$

$$+\Gamma^{ijij}_{\alpha\beta\gamma\delta}a^{+}_{i\mathbf{p}_1\alpha}a^{+}_{j\mathbf{p}_2\beta}a_{i\mathbf{p}_3\gamma}a_{j\mathbf{p}_4\delta}+(i\to j)]\,. \tag{3}$$

Γ is the bare vertex part:

$$\Gamma^{ijkl}_{\alpha\beta\gamma\delta}=\langle i\mathbf{p}_1\alpha j\mathbf{p}_2\beta|k\mathbf{p}_3\gamma l\mathbf{p}_4\delta\rangle\delta_{\alpha\delta}\delta_{\beta\gamma}-\langle i\mathbf{p}_1\alpha j\mathbf{p}_2\beta|l\mathbf{p}_4\delta k\mathbf{p}_3\gamma\rangle\delta_{\alpha\gamma}\delta_{\beta\delta}\,, \tag{4}$$

with

$$\langle i\mathbf{p}_1\alpha j\mathbf{p}_2\beta|k\mathbf{p}_3\gamma l\mathbf{p}_4\delta\rangle$$

$$=\int d\mathbf{r}_1d\mathbf{r}_2\phi^{*}_{j\mathbf{p}_1\alpha}(\mathbf{r}_1)\phi^{*}_{j\mathbf{p}_2\beta}(\mathbf{r}_2)V(\mathbf{r}_1,\mathbf{r}_2)\phi_{k\mathbf{p}_3\gamma}(\mathbf{r}_2)\phi_{l\mathbf{p}_4\delta}(\mathbf{r}_1)\,, \tag{5}$$

and $a^{+}_{i\mathbf{p}\sigma}(a_{i\mathbf{p}\sigma})$ is the creation (annihilation) operator corresponding to the excitation of electrons (or holes) in ith band with spin σ and momentum $\mathbf{p}$. μ is the chemical potential. $\phi^{*}_{i\mathbf{p}\sigma}$ is a single-particle wave-function. Here, we suppose that the vertex function (3) consists of the effective interactions between the carriers caused by the linear vibronic coupling in the several bands and the screened coulombic interband interaction of the carriers.

When we use the two-band Hamiltonian (1) and define the order parameters for the singlet exciton, triplet exciton, and singlet Cooper pair, the mean field Hamiltonian can be easily derived.[25−29,33−36] We will forcus on four electron scattering processes:

$$g_1=\langle ii|ii\rangle=\langle jj|jj\rangle\,, \tag{6}$$

$$g_2=\langle ii|jj\rangle=\langle jj|ii\rangle\,, \tag{7}$$

$$g_3=\langle ij|ij\rangle=\langle ji|ji\rangle\,, \tag{8}$$

$$g_4=\langle ij|ji\rangle=\langle ji|ij\rangle\,. \tag{9}$$

Here, g_1 and g_2 represent the intraband two-particle normal and umklapp scatterings, respectively. g_3 denotes the interband two-particle umklapp process. g_4 indicates the interband two-particle interaction on different bands (see Fig. 1). Note that Γ's are given by

$$\begin{aligned}\Gamma^{iiii}_{\alpha\beta\gamma\delta}&=\Gamma^{jjjj}_{\alpha\beta\gamma\delta}=g_1(\delta_{\alpha\delta}\delta_{\beta\gamma}-\delta_{\alpha\gamma}\delta_{\beta\delta})\,,\\ \Gamma^{iijj}_{\alpha\beta\gamma\delta}&=\Gamma^{jjii}_{\alpha\beta\gamma\delta}=g_2(\delta_{\alpha\delta}\delta_{\beta\gamma}-\delta_{\alpha\gamma}\delta_{\beta\delta})\,,\\ \Gamma^{ijij}_{\alpha\beta\gamma\delta}&=\Gamma^{jiji}_{\alpha\beta\gamma\delta}=g_3\delta_{\alpha\beta}\delta_{\beta\gamma}-g_4\delta_{\alpha\gamma}\delta_{\beta\delta}\,,\end{aligned} \tag{10}$$

where the antisymmetrized vertex function Γ is considered to be a constant independent of the momenta. The spectrum is elucidated using the Green function method. From Green's functions that characterize the CDW, SDW, and SSC phases, we obtain a self-consistent equation according to the traditional procedure,[25−29,33−36] from which we can obtain expressions of the transition temperature.

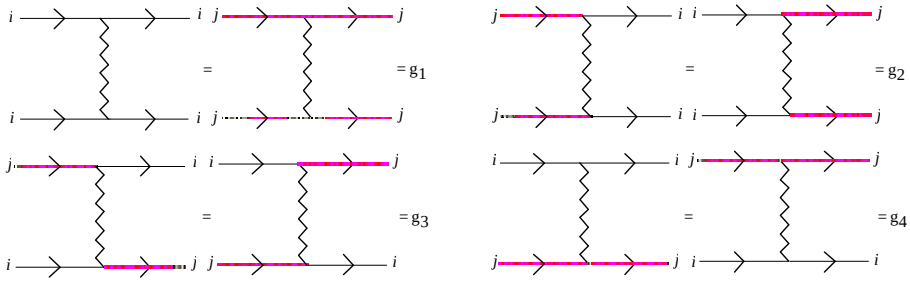

Fig. 1. Electron–electron interactions. Dependence of g on the direction of momentum space is ignored in this model, i.e. $g_x(\mathbf{k}) \approx g_x(x = i, j)$. We assume that g_x is constant.

In the framework of the one-band model, the electronic phases are characterized by

$$-g_2 - 2g_3 + g_4 > 0, \qquad \text{for CDW} \tag{11}$$

$$g_2 + g_4 > 0, \qquad \text{for SDW} \tag{12}$$

$$-g_1 > 0, \qquad \text{for SSC} \tag{13}$$

In the framework of the two-band model, we have already derived the expressions of the transition temperature for CDW, SDW, and SSC. Earlier,[25,27] we have investigated the dependence of T_c on the hole and the electron concentration for superconductivity of copper oxides using the two-band model and have obtained a phase diagram of $Bi_2Sr_2Ca_{1-x}Y_xCu_2O_8$ (Bi-2212) by means of the above expressions for transition temperature. The dependence of T_c on Δp can be reproduced in good agreement with the experiment.[37] Recently, we have also calculated the phase diagrams of copper oxides, Anthracene, Oligothiophene and C_{60} crystals.[28]

2.2. *Two-particle Green's function*

In this subsection, we will use the two-particle Green's function[18–20] to investigate the physical properties of superconductivity in a two-band model. Firstly, let us define the one-particle Green's functions:

$$G^{\nu}_{\sigma}(\mathbf{k}, t' - t) = \langle -iT[a_{\nu\mathbf{k}\sigma}(t)a^{+}_{\nu\mathbf{k}\sigma}(t')]\rangle\,, \tag{14}$$

where σ and ν label the spin and the band, respectively. The Green's function derived using the two-band model of (1) is

$$i\frac{\partial}{\partial t}\langle -iT[a_{\nu\mathbf{k}\sigma}(t)a^{+}_{\nu\mathbf{k}\sigma}(t')]\rangle = \delta(t - t') + \left\langle -iT\left[i\frac{\partial a_{\nu\mathbf{k}\sigma}(t)}{\partial t}a^{+}_{\nu\mathbf{k}\sigma}(t')\right]\right\rangle\,, \tag{15}$$

where

$$i\frac{\partial a_{\nu\mathbf{k}\sigma}(t)}{\partial t} = [a_{\nu\mathbf{k}\sigma}, H]\,. \tag{16}$$

Equation (15) can be rewritten, after the insertion of (16), as

$$
\begin{aligned}
&i\frac{\partial}{\partial t}\langle -iT[a_{\nu\mathbf{k}\sigma}(t)a^{+}_{\nu\mathbf{k}\sigma}(t')]\rangle \\
&\quad = \delta(t-t') + (\varepsilon_\nu - \mu)\langle -iT[a_{\nu\mathbf{k}\sigma}(t)a^{+}_{\nu\mathbf{k}\sigma}(t')]\rangle \\
&\qquad + \frac{1}{2}\sum_{\beta\gamma\delta}\sum_{\delta(\mathbf{k}+\mathbf{p}_2,\mathbf{p}_3+\mathbf{p}_4)} g_1(\delta_{\sigma\delta}\delta_{\beta\gamma} - \delta_{\sigma\gamma}\delta_{\beta\delta})G^{\gamma\delta\beta\sigma}_{2\nu\nu\nu\nu}(\mathbf{p}_3,\mathbf{p}_4,\mathbf{p}_2,\mathbf{k};t,t') \\
&\qquad + \frac{1}{2}\sum_{\nu'}\sum_{\beta\gamma\delta}\sum_{\delta(\mathbf{k}+\mathbf{p}_2,\mathbf{p}_3+\mathbf{p}_4)} g_2(\delta_{\sigma\delta}\delta_{\beta\gamma} - \delta_{\sigma\gamma}\delta_{\beta\delta})G^{\gamma\delta\beta\sigma}_{2\nu'\nu'\nu\nu}(\mathbf{p}_3,\mathbf{p}_4,\mathbf{p}_2,\mathbf{k};t,t') \\
&\qquad + \frac{1}{2}\sum_{\nu'}\sum_{\beta\gamma\delta}\sum_{\delta(\mathbf{k}+\mathbf{p}_2,\mathbf{p}_3+\mathbf{p}_4)} (g_3\delta_{\sigma\delta}\delta_{\beta\gamma} - g_4\delta_{\sigma\gamma}\delta_{\beta\delta})G^{\gamma\delta\beta\sigma}_{2\nu\nu'\nu'\nu}(\mathbf{p}_3,\mathbf{p}_4,\mathbf{p}_2,\mathbf{k};t,t')\,,
\end{aligned}
\tag{17}
$$

where

$$
G^{\gamma\delta\beta\sigma}_{2\nu\nu\nu\nu}(\mathbf{p}_3,\mathbf{p}_4,\mathbf{p}_2,\mathbf{k};t,t') = \langle -iT[a_{\nu\mathbf{p}_3\gamma}(t)a_{\nu\mathbf{p}_4\delta}(t)a^{+}_{\nu\mathbf{p}_2\beta}(t-0)a^{+}_{\nu\mathbf{k}\sigma}(t')]\rangle\,, \tag{18}
$$

$$
G^{\gamma\delta\beta\sigma}_{2\nu'\nu'\nu\nu}(\mathbf{p}_3,\mathbf{p}_4,\mathbf{p}_2,\mathbf{k};t,t') = \langle -iT[a_{\nu'\mathbf{p}_3\gamma}(t)a_{\nu'\mathbf{p}_4\delta}(t)a^{+}_{\nu\mathbf{p}_2\beta}(t-0)a^{+}_{\nu\mathbf{k}\sigma}(t')]\rangle\,, \tag{19}
$$

$$
G^{\gamma\delta\beta\sigma}_{2\nu\nu'\nu'\nu}(\mathbf{p}_3,\mathbf{p}_4,\mathbf{p}_2,\mathbf{k};t,t') = \langle -iT[a_{\nu\mathbf{p}_3\gamma}(t)a_{\nu'\mathbf{p}_4\delta}(t)a^{+}_{\nu'\mathbf{p}_2\beta}(t-0)a^{+}_{\nu\mathbf{k}\sigma}(t')]\rangle\,. \tag{20}
$$

Here, ν' indicates band different from ν. To calculate the density of electron states, we have to focus on the case of $t' \to t-0$. The two-particle Green functions in (17) is rewritten as $G_2(\mathbf{p}_3,\mathbf{p}_4,\mathbf{p}_2,\mathbf{k};t-t')(t' \to t-0)$. In this study, we investigate only the spectral properties of the two-particle Green's function for superconductivity. Therefore, we have

$$
G^{\gamma\delta\beta\sigma}_{2\nu\nu\nu\nu}(\mathbf{p}_3,\mathbf{p}_4,\mathbf{p}_2,\mathbf{k};t,t') = \langle -iT[a_{\nu\mathbf{p}_3\gamma}(t)a_{\nu\mathbf{p}_4\delta}(t)a^{+}_{\nu\mathbf{p}_2\beta}(t')a^{+}_{\nu\mathbf{k}\sigma}(t')]\rangle\,. \tag{21}
$$

For simplicity, we consider only three cases: (1) $g_1 \neq 0$ and others $= 0$, (2) $g_2 \neq 0$ and others $= 0$, and (3) $g_1 \neq 0$, $g_2 \neq 0$ and others $= 0$.

2.2.1. *Traditional superconductivity*

In general, in the framework of BCS theory, the Hamiltonian is described by a single-band model. In the effective e–e interaction of (1), we consider $g_1 \neq 0$ and others $= 0$ and focus only on the single-band model. According to the approach[21] used for phonon system which is based on the method of Bogolubov–Tyablikov,[38,39] we can derive the two-particle electron Green's function. The spectral features of the electronic system are described by the Fourier component of this function. The Fermi level for the simplest case of one-electron zone crossing are given as

$$
G^{\gamma\delta\beta\sigma}_{2\nu\nu\nu\nu}(\mathbf{p}_3,\mathbf{p}_4,\mathbf{p}_2,\mathbf{k};t-t') = \frac{f(\mathbf{k},\mathbf{k}',\omega)\sum_{\sigma,\sigma'}\phi(\sigma,\sigma')}{1 - g_1\sum_{\mathbf{q}}K(\omega,\mathbf{k},\mathbf{k}',\mathbf{q})}\,, \tag{22}
$$

where

$$K(\omega, \mathbf{k}, \mathbf{k}', \mathbf{q}) = \frac{2 - n^{\nu}_{\mathbf{k}+\mathbf{q}} - n^{\nu}_{\mathbf{k}'-\mathbf{q}}}{2\omega - \varepsilon^{\nu}_{\mathbf{k}+\mathbf{q}} - \varepsilon^{\nu}_{\mathbf{k}'-\mathbf{q}}} \tag{23}$$

$n_{\mathbf{k}}$ indicates the filling number of electrons and g_1 is the effective Fourier component of e–e interaction. If the coupling constant of e–e interaction (renormalized by electron–phonon interaction) becomes negative, coupled states will appear in the electronic system. In previous papers,[18–20] we have presented the analysis of the spectral properties of two-particle Green function. According to the same procedure,[18–20] we obtain the equation for coupled states in the electronic system:

$$1 - g_1 N(\varepsilon_f) \left[\ln \left| 1 - \frac{\Delta}{a} \right| \right] = 0 \,, \tag{24}$$

where

$$N(\varepsilon_f) = \sqrt{2}\pi m^*_{\nu} \sqrt{m^*_{\nu}\varepsilon}(2 - n^{\nu}_{\mathbf{k}+\mathbf{q}} - n^{\nu}_{\mathbf{k}'-\mathbf{q}})|_{\varepsilon=\varepsilon_f} \,, \tag{25}$$

$$n^{\nu}_{\mathbf{k}} = \frac{1}{\exp\left(\dfrac{\varepsilon^{\nu}_{\mathbf{k}} - \varepsilon_f}{T}\right) + 1} \,, \tag{26}$$

$a = 2(\omega - \varepsilon_f - \Delta_{\nu} - E), E = k^2/2m, \varepsilon = q^2/2m$, and $m^*_{\nu} = m_{\nu}/m$. Here m^*_{ν} is the reduced effective mass of electron and m is the mass of the free electron. If $g_1 < 0$, we can find solutions of (24) for superconductivity.

2.2.2. *Copper oxides*

In copper oxides, the effective e–e interaction g_2 is extremely important in the high-T_c superconductivity.[27,40] Thus, let us consider $g_2 \neq 0$ and others $= 0$. The two-particle Green function (21) is

$$G^{\gamma\delta\beta\sigma}_{2\nu\nu\nu\nu}(\mathbf{p}_3, \mathbf{p}_4, \mathbf{p}_2, \mathbf{k}; t - t') = \frac{f(\mathbf{k}, \mathbf{k}', \omega) \sum_{\sigma,\sigma'} \phi(\sigma, \sigma')}{1 - g_2^2 \sum_{\mathbf{q},\mathbf{q}'} K_2(\omega, \mathbf{k}, \mathbf{k}', \mathbf{q}, \mathbf{q}')} \,, \tag{27}$$

where

$$K_2(\omega, \mathbf{k}, \mathbf{k}', \mathbf{q}, \mathbf{q}') = \frac{(2 - n^{\nu'}_{\mathbf{k}+\mathbf{q}-\mathbf{q}'} - n^{\nu'}_{k'-q+q'})(2 - n^{\nu}_{\mathbf{k}+\mathbf{q}} - n^{\nu}_{\mathbf{k}'-\mathbf{q}})}{(2\omega - \varepsilon^{\nu'}_{\mathbf{k}+\mathbf{q}-\mathbf{q}'} - \varepsilon^{\nu'}_{\mathbf{k}'-\mathbf{q}+\mathbf{q}'})(2\omega - \varepsilon^{\nu}_{\mathbf{k}+\mathbf{q}} - \varepsilon^{\nu}_{\mathbf{k}'-\mathbf{q}})} \,. \tag{28}$$

According to the procedure given earlier,[18–20] we study the situation near the extremum (minimum or maximum) of the electron zone. We suppose $\mathbf{k} = \mathbf{k}' = \mathbf{k}_0 + \mathbf{k}''$ and $\varepsilon^{\nu}_{\mathbf{k}0} = \varepsilon^{\nu}$ corresponds to the extremum of zone. We expand the energy in terms of momentum $\mathbf{k} \pm \mathbf{q}$ up to second order and suppose that the energy extremum is located near the Fermi level. Then, the sum in the denominator of (27) can be reduced to the following approximate expression:

$$\sum_{\mathbf{q},\mathbf{q}'} K_2(\omega, \mathbf{k}, \mathbf{k}', \mathbf{q}, \mathbf{q}') \approx N(\varepsilon^{\nu}_f) N(\varepsilon^{\nu'}_f) \ln \left| \left(1 - \frac{2\Delta}{a_{\nu'}}\right) \left(1 - \frac{2\Delta}{a_{\nu}}\right) \right| \,. \tag{29}$$

Thus, we obtain the equation for coupled states in the electronic system:

$$1 - g_2^2 N(\varepsilon_f^{\nu}) N(\varepsilon_f^{\nu'}) \ln \left| \left(1 - \frac{2\Delta}{a_{\nu'}}\right) \left(1 - \frac{2\Delta}{a_{\nu}}\right) \right| = 0\,. \tag{30}$$

From (30), $g_2 \neq 0$ we can find the possibility of a solution for coupled states. Thus, the effective e–e interaction g_2 with positive value contributes to superconductivity.

2.2.3. *Cooperative mechanism*

Here, we consider that $g_1 \neq 0, g_2 \neq 0$, and others $= 0$. In a similar way, the two-particle Green function of (21) is approximately equal to

$$G_{2\nu\nu\nu\nu}^{\gamma\delta\beta\sigma}(\mathbf{p}_3, \mathbf{p}_4, \mathbf{p}_2, \mathbf{k}; t - t') = \frac{f(\mathbf{k}, \mathbf{k}', \omega) \sum_{\sigma,\sigma'} \phi(\sigma, \sigma')}{[1 - (g_1 + g_2) \sum_{\mathbf{q}} K(\omega, \mathbf{k}, \mathbf{k}', \mathbf{q})][1 - (g_1 - g_2) \sum_{\mathbf{q}} K(\omega, \mathbf{k}, \mathbf{k}', \mathbf{q})]}\,, \tag{31}$$

where $K(\omega, \mathbf{k}, \mathbf{k}', \mathbf{q})$ is given by (23). The sum in the denominator of (31) is performed in a similar way, and the equation for coupled states in the electronic system is approximately equal to

$$\left[1 - (g_1 + g_2) N(\varepsilon_f) \ln \left|1 - \frac{\Delta}{a}\right|\right] \left[1 - (g_1 - g_2) N(\varepsilon_f) \ln \left|1 - \frac{\Delta}{a}\right|\right] = 0\,. \tag{32}$$

From here, when $g_1 + g_2 < 0$ or $g_1 - g_2 < 0$, we can again find the solutions.

3. Discussion

In the previous section, we have approximately calculated two-particle Green's functions for three cases, which are the traditional superconductivity, the copper oxides superconductor, and superconductivity due to cooperative mechanism, in the framework of a two-band model. From these Green's functions, we have derived the equation for coupled states.

In the case of a single-band model, which indicates traditional superconductivity such as BCS theory, it is necessary that effective e–e interaction is negative ($g_1 < 0$) for the realization of superconductivity. The maximal transition temperature for superconductivity predicted by the theory is about 40 K. On the other hand, in a two-band model with negative g_1, we can expect that the transition temperature becomes higher than that derived from the single-band model, because of tunneling of Cooper pair between two bands. The tunneling of Cooper pair stabilizes the order parameter of superconductivity.[41,42]

In the framework of a two-band model, we consider that the Fermi level crosses with two bands. The results derived from the two-particle Green's function in the previous section suggested that superconductivity appears for $g_2 < 0$ or $g_2 > 0$. Note that g_2 contributes to SDW. From the results with nonzero g_1 and g_2 (cooperative mechanism), we expect T_c to be higher than the that of copper oxides.

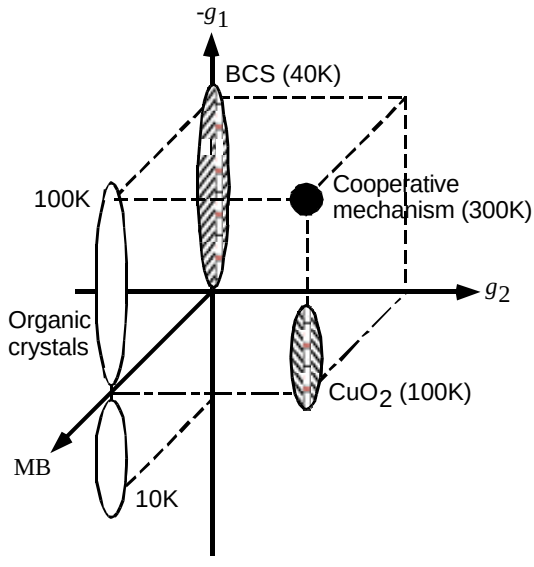

Fig. 2. Schematic diagram for superconductivity. MB means many-band effects.

From these results, we present a schematic diagram for superconductivity as shown in Fig. 2. Superconductivity of MgB_2 may arise from multi-band effects. The mechanism of High-T_c superconductivity such as that in the copper oxides may be close to a cooperative mechnism. We calculated the two-particle Green's function in a two-band model and derive the equation for coupled states. In the framework of a two-band model, our results predict that superconductivity appears, even if e–e interaction is positive. We can expect that the transition temperature is higher than that of the copper oxides.

Acknowledgment

H.N is grateful for a financial support of the Ministry of Education, Science and Culture of Japan (Research No. 12020234, 13740328). S.K. is greatful to INTAS (654). The authors thank Profs. S. Aono, M. Kimura, and K. Nishikawa, Drs. H. Kawabe and K. Ohta for their continued encouragement helpful discussion.

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