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Article *in* physica status solidi (b) · October 1990

DOI: 10.1002/pssb.2221610229

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phys. stat. sol. (b) **161**, 745 (1990)

Subject classification: 74.20; 71.45

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Superconducting Current in a Bisoliton Superconductivity Model

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It is shown that the transition into a superconducting state with the current which is described by a bisoliton superconductivity model is accompanied by the deformation of the spectrum of one-particle states of the current carriers. The deformation value is proportional to the conducting current force. The residuary resistance is absent in such states.

Показано, что переход в сверхпроводящее состояние с током, описываемое бисолитонной моделъю сверхпроводимости, сопряжен с деформацией спекрта одночастичных состояний тока. Величина деформации пропорционалъна силе сверхпроводящего тока. В таких состояниях отсутствует остаточное сопротивление.

1. Introduction

According to the concepts of BCS superconductivity theory [1, 2] the superconducting current is generated by the motion of Cooper pairs with constant velocities. In a single-particle formulation this corresponds to the Fermi sphere displacement by the Cooper pair momentum value in the momentum space. However, such a view on the current in Fermi systems has some essential drawbacks [3]. First, this is the violation of the Pauli principle. Indeed, within this approach under the directed motion of electrons (holes) generating current in the system, the possible excitation of electrons being in the low-energy states is assumed implicitly. This leads to the fact that all electrons which are in the system including those not participating in the Cooper pairs formation, take part in the current production. At temperatures lower than the superconducting transition (but nonzero) some electrons near the Fermi surface will be unpaired and, thus, will experience a resistance under directed motion (the so-called residuary resistance [4]). Such a resistance in BCS theory is suppressed but nonzero. This leads naturally to deceleration of the whole electron system and, consequently, of the superconducting condensate.

Here we show that in the bisoliton superconductivity model [5] there exist superconducting states of an electron (hole) system with currents devoid of such deficiencies. A direct result of such states of the superconductor is the asymmetry of the current–voltage dependences in tunneling systems with respect to the sign of the applied field (the dependence of the superconducting transition dielectric gap value on the direction of current in tunneling metal-superconductor systems). At low temperatures and small tunneling contact resistance the current dependence on the external potential in the region of values less than the gap width is linear. This was experimentally observed in [7, 8].

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2. Deformation Field

According to the bisoliton superconductivity theory [5, 6] the transition to a superconducting state is due to the appearance of a bisoliton condensate accompanied by the formation of a local deformation field. This field is described by the potential

$$W(x) = -2Jg\phi^2(x) . \tag{2.1}$$

The constants J and g are determined, respectively, by

$$J = \frac{\hbar^2}{2ma^2}, \qquad g = \frac{\sigma^2}{2J\varkappa(1-s^2)}, \qquad s = \frac{v}{c} .$$

Here a is the chain period, σ the deformational interaction parameter, $\varkappa$ the elasticity coefficient, m the effective mass of current carriers (electrons or holes), v the bisoliton condensate velocity, and c the sound velocity in the system.

The function $\phi(x)$ is periodic with the period aL (L^{-1} is the bisoliton concentration) and normalized by

$$\int_0^L \phi^2(x)\, dx = 1 , \qquad \phi(x + aL) + \phi(x) . \tag{2.2}$$

$\phi(x)$ is the solution to the nonlinear equation

$$\left[J\frac{d}{dx} - W(x) + E_s \right] \phi(x) = 0 , \tag{2.3}$$

where E_s is the bisoliton generation energy.

The solution of (2.3) satisfying the conditions (2.2) can be represented by the Jacobi elliptic functions dn (u, q),

$$\phi(x) + \sqrt{\frac{g}{2}}\, \frac{dn(u, q)}{E(q)}, \qquad u = \frac{gx}{E(q)} . \tag{2.4}$$

The modulus q is determined by

$$gL = 2K(q)\, E(q) \tag{2.5}$$

involving total elliptic integrals, $K(q)$ and $E(q)$, of the first and second kinds, respectively [11].

The deformation field (2.1) is, thus, a system of periodically distributed potential wells (with a period aL) each of which has a quasi-particale pair with opposite momenta and spins. It is illustrated in Fig. 1. When a bisoliton produced in the deformation well due to pairing of quasi-particles with momenta

$$p_1 = -\hbar k_F , \qquad p_2 = \hbar k_F + \hbar Q , \tag{2.6}$$

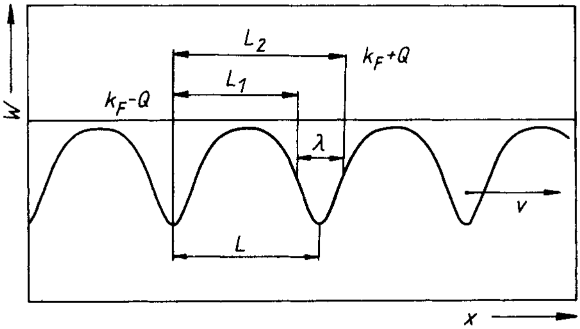

Fig. 1. The deformation field $W(x)$ with period L moving with a speed $v = \hbar Q/2m$. L_1, L_2 are the periods of the deformation field for quasi-particles with impulses k_F-Q and k_F+Q, respectively

moves with velocity $v = \hbar Q/2m$, where Q is the impulse of the bisoliton, the deformation field moves with the same velocity [8]. Such moving field in a coordinate system related to it is described by (2.1) where it is necessary to replace

$$x \to \xi = x - vt, g \to \frac{g}{\left(1 - \frac{v^2}{c^2}\right)}, \qquad \hbar k' = \hbar k + \frac{Q\hbar}{2}. \tag{2.7}$$

The energy of one-particle states $\varepsilon(k) = Jk^2a^2$ equals

$$\varepsilon(k) \Rightarrow \varepsilon\left(k - \frac{Q}{2}\right) = \varepsilon'(k'). \tag{2.8}$$

The formation of the bisoliton deformation field results in rearranging the energy spectrum of one-particle states.

3. One-Particle States

The wave function $\psi(\xi, t)$ of one-particle stationary states in the coordinate system moving with a bisoliton condensate is a solution of the Schrödinger equation with the potential $W(\xi)$ determined by (2.1),

$$\left(J\frac{\mathrm{d}}{\mathrm{d}\xi} - W(\xi) + E + E_F\right)\psi(\xi) = 0, \tag{3.1}$$

where $E_F = Jk_F^2a^2$ is the Fermi energy.

To determine the one-particle spectrum we write $\psi(\xi)$ in the form of an expansion in terms of the functions $\varphi_k(\xi)$,

$$\psi(\xi) = \sum_k u(k)\, \varphi_k(\xi), \tag{3.2}$$

$$\varphi_k(\xi) = \frac{1}{\sqrt{l}} \exp(ik\xi). \tag{3.3}$$

The functions $\psi(\xi)$ and $\varphi_k(\xi)$ are normalized in the space $l \to \infty$ where l is the chain length, whence the relation follows:

$$\sum_k |u(k)|^2 = 1 . \tag{3.4}$$

Using (3.2) and (3.3) we write (3.1) as

$$[-\varepsilon(k) + E + E_F]\, u(k) + \sum_{k_1} W(k - k_1)\, u(k_1) = 0 , \tag{3.5}$$

where

$$W(k - k_1) = \int\limits_{-(l/2)}^{l/2} \varphi_k^*(\xi)\, W(\xi)\, \varphi_k(\xi)\, d\xi . \tag{3.6}$$

Equation (3.5), with the use of periodic properties of the potential $W(\xi)$,

$$W(q) = U(q) \frac{L}{l} \sum_{n=-(l/2L)}^{l/2L} \exp(-iLng) , \tag{3.7}$$

where

$$U(q) = \frac{1}{L} \int\limits_{-(L/2)}^{L/2} W(\xi) \exp(-iQ\xi)\, d\xi , \tag{3.8}$$

can be simplified.

$W(q)$ is nonzero when

$$Lq = 2\pi\nu; \qquad \nu = \pm 1, \pm 2, \pm 3, \ldots \tag{3.9}$$

For quasi-particles with momenta (3.6) making up the moving bisoliton (3.9) takes the form

$$L(2k_F + Q) = \pm 2\pi\nu . \tag{3.10}$$

Depending on the k_F and Q ratio the equality (3.10) results in two values L. For definiteness we choose $Q > 0$. We get then from (3.10)

$$\begin{aligned} &L_1(-2\,|k_F| + Q = 2\pi\nu , \\ &L_2(2\,|k_F| + Q) = 2\pi\nu , \end{aligned} \tag{3.11}$$

where $L_{1,2} = L \pm (\lambda/2)$ and $Q \ll |k_F|$. The parameter λ characterizes the effective change in the distance between the wells due to changing the quasi-particle motion velocity when they interact through the deformation field and is determined from

$$|k_F|\, \lambda - QL = 0 .$$

Thus, for $L_{1,2}$ we get

$$L_{1,2} = L \pm \frac{QL}{2\,|k_F|} .$$

Fig. 1 shows the moving system of periodically distributed wells $W(\xi)$ and the change in effective periods of quasi-particles with different momenta forming a bisoliton. Taking into account (3.11) we get

$$|k_F|\, L\left(1 - \frac{Q^2}{4k_F^2}\right) = \pi\nu\,. \tag{3.12}$$

Equation (3.12) establishes a relation between the bisoliton density L^{-1} and that of Fermi particles proportional to k_F. Thus, according to (3.5), the deformation field hybridized states with the wave numbers $|k| < |k_F|$ and the states with $|k| > |k_F|$. For the states with $k > 0$ hybridization concerns the states displaced by the wave vector $-2k_F + Q$. The states with $k < 0$ are hybridized with the states displaced by the wave vector $2k_F + Q$. Equation (3.5) will then be written as

$$\begin{cases} [-\varepsilon(k) + E + E_F]\, u(k) + v(k)\, \Delta = 0\,, \\ \Delta v(k) + [-\,\varepsilon(k - 2k_F - Q) + E + E_F]\, v(k) = 0\,. \end{cases} \tag{3.13}$$

The amplitudes $u(k)$ and $v(k) = u(k - 2k_F - Q)$ satisfy the normalization condition

$$|u(k)|^2 + |v(k)|^2 = 1\,. \tag{3.14}$$

The parameter Δ is defined by the expression

$$\Delta = U(2k_F)\,.$$

From the system (3.13) we get the expression for the energy of a particle in the stationary coordinate system

$$E_\pm(k) = \tfrac{1}{2}\,[\varepsilon(k) + \varepsilon(k - 2k_F - Q) - 2E_F] \pm \sqrt{\tfrac{1}{4}\,[\varepsilon(k) - \varepsilon(k - 2k_F - Q)]^2 + \Delta^2}\,. \tag{3.15}$$

Analogously we get the value for the amplitudes $u(k)$ and $v(k)$ as functions of the wave numbers,

$$\begin{Bmatrix} u^2(k) \\ v^2(k) \end{Bmatrix} = \frac{1}{2}\left[1 \mp \frac{\varepsilon(k) - \varepsilon(k - 2k_F - Q)}{\sqrt{[\varepsilon(k) - \varepsilon(k - 2k_F - Q)]^2 + 4\Delta^2}}\right]. \tag{3.16}$$

Equations (3.15) and (3.16) are valid at $k_F > 0$. As follows from (3.15) in the one-particle spectrum the excitations form a gap. Its position depends on the value Q, i. e. the direction and the force of the running current. Fig. 2 shows the energy structure of the one-particle energy spectrum in a superconducting state with current. For energies near E_F we get from (3.15) and (3.16) the following expressions for the one-particle state energy $E(k)$ and amplitudes $u(k)$ and $v(k)$:

$$E(k) = Jk_F Q a^2 \pm \sqrt{4E_F\varepsilon\left(k - k_F - \frac{Q}{2}\right) + \Delta^2}\,, \tag{3.17}$$

$$\begin{Bmatrix} u^2(k) \\ v^2(k) \end{Bmatrix} = \frac{1}{2}\left\{\mp \frac{2Jk_F\left(k - k_F - \frac{Q}{2}\right)a^2}{\sqrt{4E_F\varepsilon\left(k - k_F + \frac{Q}{2}\right) + \Delta^2}}\right\}. \tag{3.18}$$

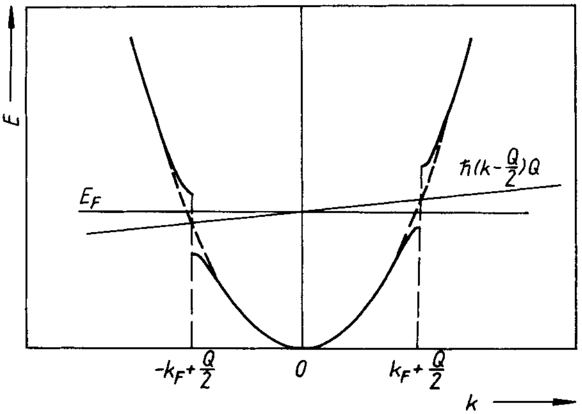

Fig. 2. The energy structure of the one-particle spectrum of bisoliton in superconducting state with current ($Q \neq 0$)

Accordingly, the wave function of one-particle states $\psi(\xi)$ in the stationary coordinate system will have the following from for the lower band:

$$\psi_{\rm H}(\xi) = \sqrt{\frac{1}{l}} \exp(ika\xi) [\pi(\varkappa) + v(\varkappa) \exp\{i(2k_{\rm F} + Q) a\xi\}], \tag{3.19}$$

and for the upper band:

$$\psi_{\rm b}(\xi) = \sqrt{\frac{1}{l}} \exp(ika\xi) [v(k) - u(k) \exp\{i(2k_{\rm F} + Q) a\xi\}], \tag{3.20}$$

The states of the lower band are occupied, of the upper band free.

4. Dielectric Gap

In a superconductor with current ($Q \neq 0$) the energy spectrum is deformed and the symmetry with respect to momentum sign inversion is broken (see Fig. 2). Such an a symmetric distribution of electrons results in the appearance of undamped superconducting current I_s equal to

$$I_s = \sum_k \frac{e\hbar}{2mi} \left(\psi_k \frac{\partial \psi_k^*}{\partial \varkappa} - \psi_k^* \frac{\partial \psi_k}{\partial \varkappa} \right), \tag{4.1}$$

where the wave function $\psi_k(\varkappa)$ is determined by (3.19). This current oscillates in time with frequency $\omega = \hbar k_{\rm F} Q/m$. Its mean value, by (4.1) and (3.19), equals

$$\langle I_s \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-(T/2)}^{T/2} I(t)\, {\rm d}t = \frac{evk_{\rm F}}{\pi} \left(1 - \frac{\Delta}{E_{\rm F}} \right). \tag{4.2}$$

With increasing superconducting current proportional to the velocity of bisoliton motion the lower band top (filled by electrons) will rise and the upper band bottom (free) will get lower. When they level, part of electrons will occupy the nonfilled band and a resistance will arise in the system. Consequently, the current may run through such a system without such resistance only when

$$\Delta_{\text{eff}} = E_b\left(k = -|k_F| + \frac{Q}{2}\right) - E_H\left(k = |k_F| + \frac{Q}{2}\right) > 0\,. \tag{4.3}$$

Using (3.17) and (4.3) we get

$$\Delta_{\text{eff}} = \Delta - 2\hbar k_F v > 0 \tag{4.4}$$

Thus, from (4.2) and (4.4) the expressions for critical velocity v_{cr} and the critical current $\langle I_{\text{cr}}\rangle$ follow:

$$v < v_{\text{cr}} = \frac{\Delta}{2\hbar k_F}, \tag{4.5}$$

$$\langle I_s\rangle < \langle I_{\text{cr}}\rangle = \frac{e\Delta}{2\pi\hbar}\left(1 - \frac{\Delta}{k_F}\right).$$

$2\Delta_{\text{eff}}$ characterizes the effective value of the dielectric gap in a superconductor with current. In the superconducting state of the system with current $I < \langle I_{\text{cr}}\rangle$ the current carrier scattering not breaking the energy band symmetry will be absent. This situation will also hold at finite temperatures, lower than the critical one. In this case the carriers excited thermally are in equilibrium and do not participate in the current transfer. Thus, we arrive at the two-component Fermi gas with one component being superconducting and the other normal. The directed motion of the normal component is accompanied by energy dissipation and heating release.

In the expressions for $E(k)$ and u, v the gap value is determined by (3.14) and (3.8) where L follows from (3.12). Integrating in (3.8) at large Lg we get the expression for the gap,

$$2\Delta = \frac{8k_E^2 J}{v\left(1 - \frac{Q^2}{4k_F^2}\right)}\, sh^{-1}\left(\frac{k_F\pi}{g}\right). \tag{4.6}$$

Depending on the value v, as is shown in [11], different values for the gap 2Δ are possible.

5. Current–Voltage Curves for Tunneling Current

The energy spectrum deformation by the superconducting current leads to experimentally observed peculiarities in the current–voltage characteristics of metal-isolator-superconductor systems. At the metal-dielectric-superconductor contacts the running current is determined by the transfer coefficient through the barrier P having the constant value [10, 12]

$$I = \frac{e}{2\Pi\hbar}\,\varrho \int P[f_1(E) - f_2(E)]\,\mathrm{d}\varepsilon + \langle I_s\rangle\,, \tag{5.1}$$

where $\langle I_s\rangle$ is the superconducting Josephson current component described by (4.2), ϱ the density of electrons in the metal incident on the barrier, $f_{1,2}(E)$ are the Fermi functions of electron distribution in metal and superconductor, respectively. The dependence of E on

$\varepsilon(k)$ near the Fermi energy is determined by (3.17) by means of which we transform (5.1) as

$$I = I_0 \frac{1}{\Delta} \int \frac{\mathrm{d}E|E|}{\sqrt{E^2 - \Delta^2}} \left[f_1 \left(E + 2E_\mathrm{F}q + Q\right) - f_2 \left(E + 2E_\mathrm{F}q\right)\right] + \langle I_\mathrm{s} \rangle \,, \tag{5.2}$$

where

$$q = \frac{Q}{k_\mathrm{F}} \,, \qquad I_0 = \frac{e}{2\pi\hbar} \varrho \Delta k_\mathrm{F}^2 \,,$$

φ is the bias voltage applied to the system.

At small voltages ($\varphi \ll \Delta$) the current running through the system will equal $I \approx \langle I_\mathrm{s} \rangle$. In this case the bias voltage φ falls only on metal and dielectric having the common resistance R. Thus, for $\langle I_\mathrm{s} \rangle$ we get

$$\langle I_\mathrm{s} \rangle = \frac{\varphi}{R} = \frac{ek_\mathrm{F}\hbar Q}{\pi m} \,, \tag{5.3}$$

whence it follows that

$$Q = \frac{\pi m \varphi}{\hbar k_\mathrm{F} R e} \,, \tag{5.4}$$

$$J a^2 k_\mathrm{F} Q = \alpha \varphi \,, \qquad \alpha = \frac{\hbar \pi}{2Re} \,. \tag{5.5}$$

The dependences of I (curves 1) and conductivity $\mathrm{d}I/\mathrm{d}\varphi$ (curves 2) on the bias voltage φ at temperature $k_\mathrm{B}T/\Delta = 0.1$ are shown in Fig. 3. The gap 2Δ asymmetry with respect to the sign φ is due to the presence of the term $Jk_\mathrm{F}Qa^2$ in the distribution functions of (5.2). In the temperature range $T \approx 0$ K for $\Delta_\pm$ from (3.16) at $\varphi = \Delta$ we get

$$\begin{aligned} \Delta_+ &= \Delta - Jk_\mathrm{F}Qa^2 \,, \\ \Delta_- &= \Delta + Jk_\mathrm{F}Qa^2 \,, \\ \sigma &= \Delta_+ - \Delta_- = 2Jk_\mathrm{F}Qa^2 = 2\alpha\Delta \,. \end{aligned} \tag{5.6}$$

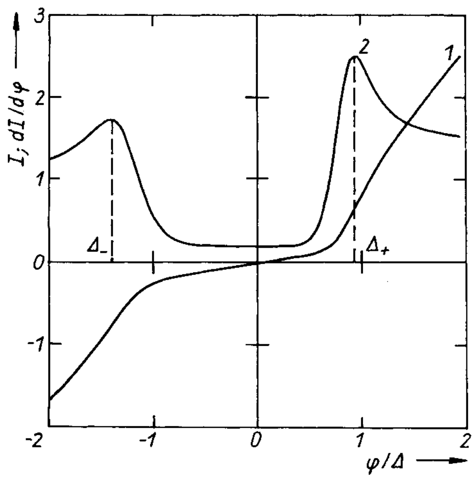

Fig. 3. (1) Current–voltage and (2) conductivity–voltage characteristics of the superconducting current

As follows from (5.6) σ depends on the resistence R. With decreasing R the value σ increases. Simultaneously, at small φ the current is linearly dependent on the bias voltage. These peculiarities are well traced in experimental papers [7, 8, 13, 14].

6. Conclusion

Thus, the superconducting current in a bisoliton superconductivity model is accompanied by the deformation of energy bands of a one-particle excitation spectrum. Such an interpretation of the motion of a superconducting condensate, unlike the traditional treatment of the motion of the whole Fermi sphere, is devoid of deficiencies connected with the violation of the Pauli principle and the problem of the residual resistance. The scenario of superconductivity which current is independent of a specific superconductivity model and can be considered in these models. It leads to a number of physical results, a symmetric current–voltage characteristics, linear dependence of current on voltage bias at voltages less than the gap energies, etc. These peculiarities are observed experimentally [7, 8, 13, 14].

Acknowledgement

The authors are indebted to A. S. Davydov for the fruitful discussions of the results obtained.

References

[1] J. R. Schriffer, Theory of Superconductivity, Fzd. Nauka, Moscow 1970 (311).
[2] R. P. Feynman, Statistical mechanics, Izd. Mir, Moscow 1975 (p. 407).
[3] J. M. Ziman, Principles of the Theory of Solids, Cambridge University Press, London 1964.
[4] C. Kuper, Adv. Phys., **8**, 1 (1959).
[5] A. S. Davydov, Nonlinearity **2**, 383 (1989).
[6] A. S. Davydov, Supercond. Sci. Technol. **2**, 153 (1989).
[7] A. I. Akimenko, N. M. Ponomarenko, V. A. Gudimenko, I. K. Yanson, P. Samuely, and P. Kus, Fiz. nizkikh Temp. **15**, 1242 (1989).
[8] I. B. Altoreder, A. P. Volodin, I. N. Makarenko, and S. M. Steeshov, Zh. eksp. teor. Fiz., Pisma **50**, 458 (1989).
[9] G. A. Korn, and J. M. Korn, Mathematical Handbook, McGraw-Hill Book Co., New York 1968.
[10] R. Tsu and L. Esaki, Appl. Phys. Letters **22**, 562 (1973).
[11] V. N. Ermakov and S. P. Kruchinin, phys. stat. sol. (b) **156**, 333 (1989).
[12] V. N. Ermakov and E. A. Ponezha, Dokl. Akad. Nauk SSSR, Ser. A, No. 6, 52 (1989).
[13] J. R. Kirtley, J. mod. Phys. B **4**, 201 (1990).
[14] I. Takeuchi, J. S. Tsai, Y. Shimakawa, T. Manako, and Y. Kubo, Physica **C158**, 83 (1989).

(Received July 23, 1990)