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CONTROLLED NOT GATE BASED ON A TWO-LAYER SYSTEM OF THE FRACTIONAL QUANTUM HALL EFFECT

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After a coordinate transformation, the wavefunction of the ground state of the fractional Quantum Hall Effect is found to be the congregation of free independent particles immune to thermal phonon disturbance. Based on the pseudo-spin states appearing in the two layer system of fractional Quantum Hall Effect, we propose a new controlled NOT gate with very large decoherence time.

Recently, quantum computing has been attracting the attention of many researchers. Many quantum computing devices have been proposed.[1] The main difficulty in practical operation of most of those devices is the existence of strong decoherence which causes error in the computing procedure. The effect appears as the decrease in the nondiagonal terms of the density matrix of the initial pure state through the interaction between the state and environment, which results in the decrease in interference.

In this paper, we point out that

(i) the wavefunction representing the quantum state of the fractional Quantum Hall Effect (FQHE)[2,3] is found, after appropriate coordinate transformation, to be the congregation of independent "free particles" with no particle–particle interaction,

(ii) the decoherence time τ_D of the "free particle" state is expected effectively to be infinity due to ignorable thermal phonon disturbance, and

(iii) the macroscopic quantum nature with particle-number definiteness ($\Delta N \approx 0$) of FQHE has excellent adaptability to the qubit device using particle number change in comparison with the superconductor device where the macroscopic quantum nature has phase definiteness ($\Delta\Theta \approx 0$). These properties reveal the possibility that FQHE provides ideal quantum computing devices having particle quantum states with $\tau_D \to \infty$.

It was shown that any quantum computer can be composed based on the "controlled NOT gate",[4,5] where the wavefunction of a quantum element is processed by control bit and target bit. We propose a controlled NOT gate using a two-layer FQHE system.

As is well known, the ground state of a FQHE system of N charged particles at filling factor $\nu = 1/m$ on an xy 2D plane is well described by the Laughlin function

$$\begin{aligned} z_j &= \frac{x_j + iy_j}{l_0}\,, \\ \Psi_m^L &= \prod_{N\geq j>k\geq 1} (z_j - z_k)^m \exp\left(-\frac{\sum_{l=0}^{N} |z_l|^2}{4}\right), \end{aligned} \tag{1}$$

where (x_j, y_j) are the 2D coordinates of the jth particle, and $l_0 = \sqrt{\hbar/M\omega_c} = \sqrt{\hbar/Q_0 B}$ is the magnetic length, where M and Q_0 are the mass and charge of the particle, and B is the magnetic field. Using the coordinate transformation[6]

$$z^+ = \frac{\sum_{j=1}^{N} z_j}{\sqrt{N}}\,,$$

$$z_{jk} = \frac{z_j - z_k}{\sqrt{N}}\,,$$

we easily rewrite (1) as

$$\begin{aligned} \Psi_m^L &= \text{const.} \times \prod_{j>k} z_{jk}^m \exp\left(-\frac{|z^+|^2 + \sum_{j>k} |z_{jk}|^2}{4}\right) \\ &= \text{const.} \times \prod_{j>k} \left[z_{jk}^m \exp\left(-\frac{|z_{jk}|^2}{4}\right)\right], \end{aligned} \tag{2}$$

where the origin is put at the center of gravity ($z^+ = 0$). The term const. $\times$ $z^m \exp(-|z|^2/4)$ is just the ground state wavefunction of a free charged particle in magnetic field $\nabla \times \mathbf{A} \equiv \mathbf{B} = (0, 0, B)$. Equation (2) shows that the N-particle Laughlin state is re-expressed to be the congregation of independent $\binom{N}{2}$ "free particles" being described by the coordinate z_{jk}, and that each state is the solution of a single particle Hamiltonian

$$H_{jk} = \frac{1}{2M}(\mathbf{p}_{jk} - Q_0 \mathbf{A})^2\,, \tag{3}$$

where $\mathbf{p}_{jk} = (\mathbf{p}_j - \mathbf{p}_k)/\sqrt{N}$.

One must note in the Laughlin state that the spread of the wavefunction of each charged particle is $\sqrt{\Delta x^2 + \Delta y^2} \sim l_0 \ll L_{2D}$ (2D xy plane size) as seen in (1), but that the spread of the wavefunction of the "free particle" is $\sqrt{\Delta x^2 + \Delta y^2} \sim \sqrt{N} l_0 \sim L_{2D}$ as seen from (2), where L_{2D} is the size of the 2D FQHE system. This shows that the wavefunction of a "free particle" spreads effectively all over the 2D plane with wave number $k \simeq 1/L_{2D} \simeq 10^{-2}$–$10^{-3}\,\mathrm{m}^{-1}$. On the other hand, the wave number of typical phonon which is effective to change the quantum state of particles may be $k_{ph} \simeq 10^8$–$10^{10}\,\mathrm{m}^{-1}$. The large difference of wave numbers may effectively eliminate the interaction between the "free particle" and phonon.

The following unique properties may provide an ideal qubit device based on the "free particles."

(i) The immunity of "free particles" to thermal phonon disturbance results in infinitely long decoherence time ($\tau_D \to \infty$).
(ii) The quantum state of a "free particle" is simply given by the independent "single particle Hamiltonian" (3) without considering the many particle effect.
(iii) Vector potential $\mathbf{A}$ is the only factor which determines the evolution of the quantum state of "free particle."

Next we propose a new controlled NOT gate based on FQHE. By subsequent layer fabrication of GaAs and AlGaAs using molecular beam epitaxy technology, we can make the multi-layer structure of a pair of two-layer FQHE systems connected by a tunnel barrier as shown in Fig. 1.

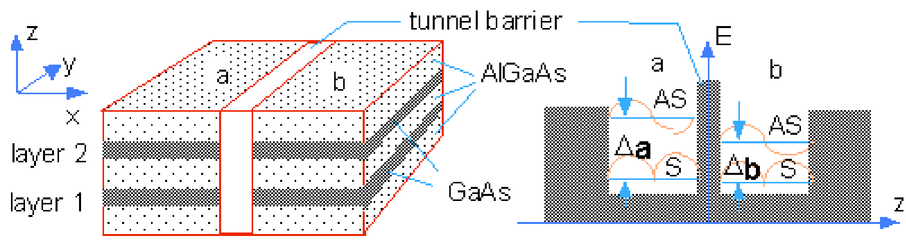

Fig. 1. A pair of two-layer FQHE systems connected by a tunneling barrier.

In the following discussion, we suppose that the all the layers are in the same FQHE state of identical filling factor ν. The upper and lower layers in each section (a or b) are connected to each other by fairly strong inter-layer tunneling described by parameter Δ_a or Δ_b in the absence of the tunneling coupling between a and b. We put $\Delta_a > \Delta_b$.

Now consider the two-layer system in section a in the absence of the a–b coupling. We write respectively the "free particle" states ψ_{1a} when the particle is in layer one and ψ_{2a} in layer two. Under an overlapping integral t, the eigenstates are given by linear combinations of ψ_{1a} and ψ_{2a},

$$\psi_{Sa} = (\psi_{1a} + \psi_{2a})\,/\sqrt{2}\,, \tag{4}$$

$$\psi_{ASa} = (\psi_{1a} - \psi_{2a})\,/\sqrt{2}\,. \tag{5}$$

Then ψ_{Sa} is more stable than ψ_{ASa} by energy $2t \equiv \Delta_a$.

It is known that (ψ_{1a}, ψ_{2a}) are respectively considered to be the "pseudo-spin" state $(|\uparrow_a\rangle, |\downarrow_a\rangle)$, and (ψ_{Sa}, ψ_{ASa}) to be $(|\longrightarrow_a\rangle, |\longleftarrow_a\rangle)$. When the device size is small and the charging energy $Q_0^2/2C_a$ is large enough, the appearance of $(|\uparrow_a\rangle, |\downarrow_a\rangle)$ states is effectively forbidden, and the system is allowed to take only the two "pseudo-spin" states $(|\longrightarrow_a\rangle, |\longleftarrow_a\rangle)$ with "Zeeman energy" separation Δ_a. Therefore, we can consider a qubit operation based on the two "pseudo-spin" states similar to the nuclear spin operation in NMR quantum computing.[7]

In order to endow the "pseudo-spin" system with the controlled NOT function, we utilize the selective tunneling coupling between a pair of the two-layer FQHE systems after the idea of dipole coupling caused by quantum confined Stark effect in two quantum dot systems.[8]

Now consider, as seen in Fig. 1, the coupling between the pair of two-layer systems through a tunneling barrier, where the coupling energy is supposed to be smaller than Δ_a and Δ_b. Since the tunneling probability is proportional to $|\langle\psi_a|H_{\rm t}|\psi_b\rangle|^2$, we know that a–b tunneling takes place only between states $|\longrightarrow\rangle_a$ and $|\longrightarrow\rangle_b$, or between $|\longleftarrow\rangle_a$ and $|\longleftarrow\rangle_b$ considering the absence of coordinate dependence of tunneling Hamiltonian $H_{\rm t}$. The tunneling coupling causes the energy-level modification in proportion to the overlapping integral between a and b. We show in Fig. 2 the change of energy levels by the introduction of the a–b tunneling coupling. We use the following state expression: $|\longrightarrow\rangle_a|\longrightarrow\rangle_b \Rightarrow |0\rangle|0\rangle$, $|\longrightarrow\rangle_a|\longleftarrow\rangle_b \Rightarrow |0\rangle|1\rangle$, $|\longleftarrow\rangle_a|\longrightarrow\rangle_b \Rightarrow |1\rangle|0\rangle$, $|\longleftarrow\rangle_a|\longleftarrow\rangle_b \Rightarrow |1\rangle|1\rangle$. When the a–b coupling is absent, the energy levels are as shown on the left of Fig. 2. When finite a–b coupling is introduced, the levels of $|0\rangle|0\rangle$ and $|1\rangle|1\rangle$ are lowered (see the right of Fig. 2). Then using the π-pulse of coherent electromagnetic waves with photon energy Δ_b', we can selectively make inversion of the states between $|1\rangle|0\rangle$ and $|1\rangle|1\rangle$, realizing the controlled NOT action similar to Ref. 8.

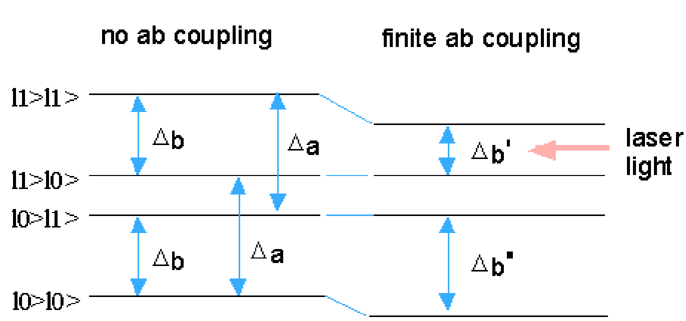

Fig. 2. Change of energy level by the introduction of tunneling coupling between a and b systems.

In conclusion, we showed that the particle state in a FQHE system has adequate quantum property to make high quality qubit for quantum computing, and proposed a new controlled NOT gate based on the state change of "pseudo-spin" appearing in a two-layer FQHE system.

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