International Journal of Modern Physics B
Vol. 32 (2018) 1840034 (14 pages)

DOI: 10.1142/S0217979218400349

Sharp-pointed susceptibility of ferromagnetic films with magnetic anisotropy inhomogeneous in thickness

Yu. I. Dzhezherya* and A. O. Khrebtov

Institute of Magnetism,
National Academy of Sciences of Ukraine
and Ministry of Education and Science of Ukraine,
36-b, Vernadsky Blvd., Kyiv 03680, Ukraine
*dui_kpi@ukr.net

D. P. Azarkh

Faculty of Physics and Mathematics,
The National Technical University
of Ukraine "Ihor Sikorsky Kyiv Polytechnic Institute",
37, Prosp. Peremohy, Kyiv 03656, Ukraine

S. P. Kruchinin

Bogolyubov Institute for Theoretical Physics,
National Academy of Sciences of Ukraine,
14b, Metrolohichna Str., Kyiv 03680, Ukraine
skruchin@i.com.ua

Accepted 7 February 2018
Published DD March 2018

It is shown that in ferromagnetic films with anisotropy inhomogeneous in thickness, an interphase boundary can be formed for certain values and directions of the magnetic field. This boundary has a stable magnetic configuration. It is oriented parallel to the film plane and separates the regions with different orientations of the magnetization. In an alternating magnetic field, the interphase boundary oscillates, which may be accompanied by a resonance. The field and frequency dependences of the components of the magnetic susceptibility tensor are determined. It is shown that the susceptibility coefficients at a resonance are extremely sensitive to the direction of the external magnetic field which can underlie the development of a highly sensitive sensor.

Keywords: Interphase boundary; ferromagnetic ferrite–garnet film; sharp-pointed susceptibility.

PACS numbers: 75.20En, 75.20Hr, 75.30Et, 75.70Cn

*Corresponding author.

1. Introduction

The magnetic properties of the monocrystalline iron garnet epitaxial films (IGEFs) grown by liquid phase epitaxy on the monocrystalline gallium gadolinium garnet (GGG) substrates differ dramatically from those of the three-dimensional monocrystalline iron garnets.[1–3]

The qualitative difference between the magnetic properties of the three-dimensional iron garnet and the IGEFs of the same composition is primarily due to a mismatch in the lattice parameters of the GGG substrate and the IGEF structure grown on it and the crystallographic orientation of the substrate, which determines the anisotropy energy.[4] In particular, IGEFs that are grown on the GGG substrates (111) have uniaxial anisotropy with the easy magnetic axis which is perpendicular to the plane of the film ($Q > 1$). The initial interest in this type of films was caused by its possibility to form cylindrical magnetic domains (CMD) used as an information-bearing medium in the storage devices.[5] Later, bismuth iron garnet films with magnetic anisotropy of the "easy-plane" type — also grown on GGG substrates (111) — found wide practical applications for visualizing a spatially inhomogeneous magnetic field in the magneto-optical nondestructive inspection.[6]

At present, monocrystalline IGEFs are widely utilized, apart from the CMD memory devices, in the magneto-optical devices to display and to process information, in the spin-wave ultra high-frequency and extremely high-frequency electronics, and as sensor elements in biomedicine. The range of applications of IGEFs is constantly expanding, which stimulates the development of novel IGEFs to meet the emerging requirements in the field of nanotechnology.

Along the indicated research line, the multilayer IGEFs have been developed. They were obtained by a sequential epitaxy of the iron garnet films with different magnetic properties. In this arrangement, each previous layer serves as a substrate for the next one.[7]

The studies of such multilayer iron garnet structures reveal that magnetic properties of the multilayer IGEFs are not an additive sum of the properties of individual layers. For example, as was shown in Ref. 8, even in the case where the iron garnet film with anisotropy "angular phase" was grown as a single layer, the nonstationary epitaxy led to a change in the film structure and to the emergence of transition layers, namely substrate-to-film and film-to-air ones, with anisotropies that are different from the anisotropy of the main layer.

While the film-to-air transition layer can be removed by etching the IGEF surface, the film-to-substrate transition layer with the "easy plane" anisotropy, which is formed at the film-substrate interface due to the entry the Ga and Gd ions into the film, cannot be removed in any way. The appearance of the film-to-substrate transition layer turns the single-layer iron garnet film to a multilayer one with "easy-plane" and "phase angle" anisotropy layers. The magnetic properties of such a multilayer IGEF are not identical to the magnetic properties of the layers it consists of. Namely, a giant anomalous susceptibility with the angular range of $\Delta\theta \leqslant 1°$ has been observed in the cases where the angle θ between the magnetic field vector $\mathbf{H}_\theta$ and the IGEF normal was equal to $\theta = 0.3$–$1.5°$, and the magnetic field was in the range $\mathbf{H}_\theta = 500$–1700 Oe corresponding to the layer of the single-domain "angle phase".

The ability to create multilayer IGEFs with a giant anomaly of the magnetic susceptibility in a narrow angular range, as was demonstrated experimentally in Ref. 8, has obtained no theoretical explanation yet, in spite of the obvious potential of this phenomenon for technological applications.

The aim of this work is to analyze the conditions essential for the creation of narrowly focused magnetic susceptibility anomalies in multilayer IGEFs, which are obtained by liquid phase epitaxy on the GGG substrates (111).

As was shown in Ref. 9 for the iron-garnet films with the Ca–Ge substitution grown on the GGG substrate (111) by liquid epitaxy, the transition layer with the "easy plane" anisotropy occurs even when the main film has an "easy axis" anisotropy and a large quality factor.

Consequently, the two-layer iron-garnet films with Gd–Ga substitution were used in the experiments. The first layer was the IGEF of the composition $(GdLu)_3(FeGa)_5O_{12}$ and with the "easy plane" anisotropy, grown on the GGG substrate (111), and the second layer was the IGEF of the composition $(YBiLuSmGd)_3(FeGa)_5O_{12}$ with the "easy axis" anisotropy and a relatively small quality factor ($Q = 1.15$–1.25) Refs. 10 and 11.

2. Experiment

The changes in the magnetostatic properties of such two-layer IGEF structure was measured by an inductive-frequency method,[12] which is a nondestructive technique and allows one to investigate the magnetic properties of individual layers of the IGEF and the two-layer IGEF as a whole.

The analysis of the experimentally measured dependences of the differential magnetic susceptibility $\boldsymbol{\chi}_d$ on the magnetic field directed perpendicularly, $\boldsymbol{\chi}_d \sim f(\mathbf{H}_\perp)$, in parallel, $\boldsymbol{\chi}_d \sim f(\mathbf{H}_\parallel)$, and at the angle $\boldsymbol{\theta}$, $\boldsymbol{\chi}_d \sim f(\mathbf{H}_\theta)$, performed as in Ref. 12, has demonstrated that the considered IGEF structure consists of the layer with the "easy plane" anisotropy and quality factor $Q < 1$, the transition layer with quality factor $0.9 < Q < 1.09$, and the layer with the "easy axis" anisotropy and quality factor $Q > 1.15$.

The "burst" of the magnetic susceptibility $\boldsymbol{\chi}$ is observed in a narrow angular range of $0 < \theta < 3°$ and reaches its maximum (11-time increase) at $\theta = 1.3°$ and $\mathbf{H}_\theta = 0.950$ kOe (Fig. 1). The deviation of the vector $\mathbf{H}_\theta$ from this direction by $\pm 30'$ leads to a change in the magnitude $\boldsymbol{\chi}$ by a factor of 5. Such a rapid change in the magnitude χ for the single-layer-film witnesses (i.e., a single-layer IGEF of the same composition which is grown under identical conditions) was not observed, indicating that the narrow angular anomaly of the differential susceptibility $\boldsymbol{\chi}_d$

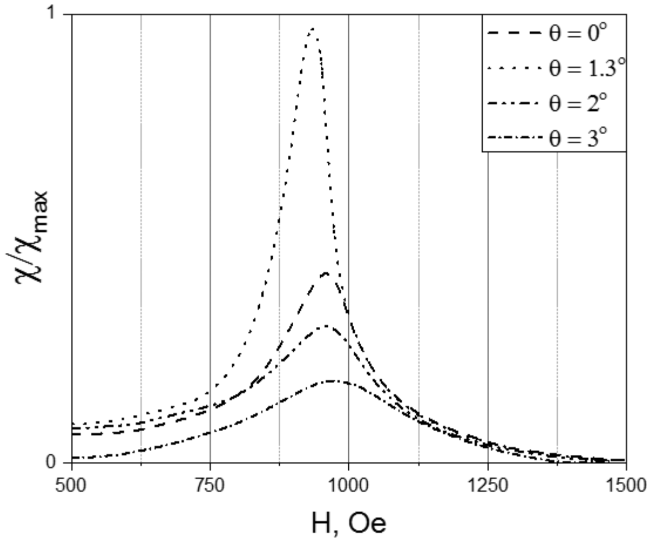

Fig. 1. Magnetic susceptibility $\boldsymbol{\chi}$ of the bilayer IGEF structure as a function of the magnetic field $\mathbf{H}_\theta$ for various values of θ.

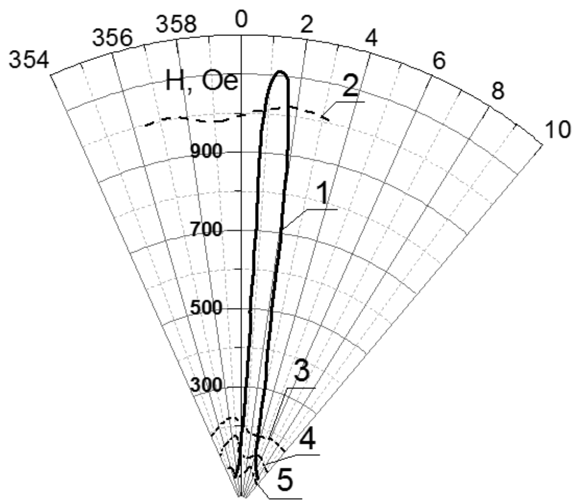

Fig. 2. Plots of $\boldsymbol{\chi}_d^{(1)} \sim f(\mathbf{H}_\theta^{(1)})$ — curve 1, $\boldsymbol{\chi}_d^{(2)} \sim f(\mathbf{H}_\theta^{(2)})$ — curve 2, $\boldsymbol{\chi}_d^{(3)} \sim f(\mathbf{H}_\theta^{(3)})$ — curve 3, $\boldsymbol{\chi}_d^{(4)} \sim f(\mathbf{H}_\theta^{(4)})$ — curve 4 for $\boldsymbol{\theta}$ changing in the range $(-1^\circ \div +3^\circ)$, where the direction of $\boldsymbol{\theta} = 0^\circ$ coincides with the normal to the film plane and crystallographic axis [111].

(Fig. 2) is solely due to the two-layer IGEF structure with the "easy plane"/"easy axis" anisotropy of the layer. Furthermore, the fact that the giant anomaly of the $\boldsymbol{\chi}_d$ susceptibility was observed in a test film with the Gd–Ga substitution, with the same orientations of the magnetic field vector $\mathbf{H}_\theta$ relative to the normal to the film plane and the crystallographic [111] axis, as a bilayer IGEF structure described in Ref. 8 with the Ca–Ge substitution, suggests that the observed anomaly is an intrinsic property of the bilayer IGEF structures with the "easy plane"/"easy axis" anisotropy of the layers.

3. Theoretical Model

To explain the observed effect, we consider the ferromagnetic film of the thickness L, whose normal coincides with the Oz axis of the coordinate system, as shown in Fig. 3. We assume that the film exhibits the uniaxial anisotropy and its direction coincides with the Oz axis. The energy density including the main types of interactions in the system is as follows:

$$W = \frac{\alpha}{2}\left(\frac{\partial \mathbf{m}}{\partial z}\right)^2 - \frac{\beta(z)}{2} m_z^2 - H_z m_z - H_x m_x + \frac{H_m^2}{8\pi}, \tag{1}$$

where α is the exchange interaction constant, $\mathbf{m}$ is film magnetization vector, $\beta(z)$ is the uniaxial anisotropy parameter, which is considered as a function of the coordinates, $\mathbf{H} = (H_x, 0, H_z)$ is an external magnetic field and $\mathbf{H}_m$ is the intrinsic magnetostatic field of the ferromagnet.

In the expression for the energy density (1), it was assumed that the magnetization distribution is homogeneous in the film plane xOy. This assumption is valid in sufficiently strong magnetic fields, where the domain structure is suppressed. The dependence of the magnetization distribution on the coordinate z is due to the modulation of the magnetic anisotropy parameter $\beta(z)$. In this case, the Maxwell's equation $\mathrm{div}\mathbf{B} = \frac{\partial}{\partial z}(H_z^m + 4\pi m_z) = 0$ implies that the magnetostatic field is uniquely determined and has the form $\mathbf{H}^m = -\mathbf{e}_z 4\pi m_z$.

In what follows, we assume that the direction of the external field slightly deviates from the normal of the film by an angle $\gamma \ll 1$, so that $\mathbf{H} = (H \sin\gamma, 0,$

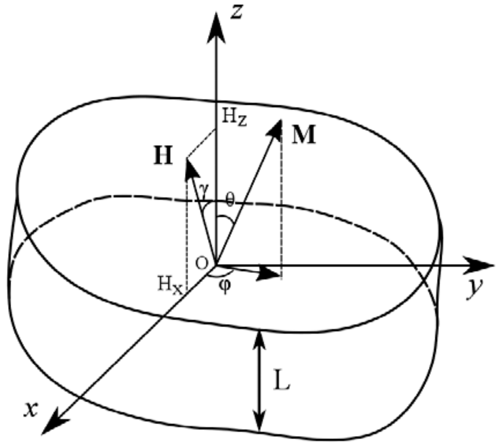

Fig. 3. Fragment of the ferromagnetic film.

$H \cos\gamma) \approx (H \cdot \gamma,\, 0,\, H)$, where H is the absolute value of the external field. This remark allows us to simplify the foregoing calculations.

The further simplification is due to the relation for the magnetization vector: $\mathbf{m}^2 = m_0^2$, where m_0 is a constant equal to the saturation magnetization of the film. Thus, it becomes possible to pass to two independent angular variables that determine the orientation of the magnetization vector of the film:

$$\mathbf{m} = m_0 \begin{pmatrix} \cos\phi \sin\theta \\ \sin\phi \sin\theta \\ \cos\theta \end{pmatrix}, \tag{2}$$

where θ and ϕ are the polar and azimuthal angles of an orientation of the magnetization vector in the coordinate system with the Oz polar axis, respectively.

In view of the assumptions and remarks given above, the energy density of the ferromagnetic system (1) in the polar coordinate system of the magnetization vector becomes

$$W = \frac{m_0^2}{2}\left\{ \alpha \left(\frac{\partial\theta}{\partial z}\right)^2 + \alpha \sin^2\theta \left(\frac{\partial\varphi}{\partial z}\right)^2 + (4\pi - \beta(z))\cos^2\theta \right. \\ \left. - 2(H/m_0)(\cos\theta - \gamma \cdot \cos\varphi \sin\theta) \right\}. \tag{3}$$

The dynamics of the magnetization of the system under consideration is determined by the L–L equation, which allows notations in the polar coordinate system and takes the processes of energy dissipation into account:

$$\begin{aligned} \frac{\partial W}{\partial\varphi} &= -\frac{m_0}{g}\sin\theta \frac{\partial\theta}{\partial t} - \lambda_2 \frac{m_0}{g}\sin^2\theta \frac{\partial\varphi}{\partial t}, \\ \frac{\partial W}{\partial\theta} &= \frac{m_0}{g}\sin\theta \frac{\partial\varphi}{\partial t} - \lambda_1 \frac{m_0}{g}\frac{\partial\theta}{\partial t}, \end{aligned} \tag{4}$$

where $g = 2\mu_0/\hbar$, μ_0 is the Bohr magneton.

Writing down the terms of Eq. (4), which are responsible for the dissipation, we consider that, in the ferrite–garnet films, the energy dissipation of moving nonlinear formations such as domain walls cannot be described by a single dissipative constant of the Gilbert–Landau–Lifshitz theory.[13] It was shown in Ref. 5 that the account for the longitudinal relaxation associated with a change in the magnetization modulus in a vicinity of the moving boundary leads to a substantial increase in the energy dissipation. This effect can easily be taken into account by formally introducing independent dissipative constants for the polar and azimuth angles.

At this stage, we specify the nature of the magnetic anisotropy to simplify the following discussion and highlight the main idea of this work. We assume that the anisotropy parameter varies slowly across the film thickness, for example linearly; so that $\beta(z) = \beta_0 + \Delta\beta \cdot (z/L)$, where $\beta_0 < 4\pi$ is some constant, and $\Delta\beta$ is the correction to define the anisotropy modulation amplitude.

In view of this notation, the L–L equation can be represented as

$$\frac{\partial}{\partial\xi}\sin^2\theta\frac{\partial\varphi}{\partial\xi} = \sin\theta\left(\gamma\, h_0\,\sin\varphi + \frac{\omega}{\omega_0}\frac{\partial\theta}{\partial\tau}\right) + \lambda_2\frac{\omega}{\omega_0}\sin^2\theta\frac{\partial\varphi}{\partial\tau}, \tag{5a}$$

$$-\frac{\partial^2\theta}{\partial\xi^2} + \sin\theta\left(h_0 - \left(1 - k\,\xi - \left(\frac{\partial\varphi}{\partial\xi}\right)^2\right)\cos\theta\right)$$
$$= \gamma h_0\cos\varphi\cos\theta + \sin\theta\left(\frac{\omega}{\omega_0}\frac{\partial\varphi}{\partial\tau} - h_1\cos\tau\right) - \lambda_1\frac{\omega}{\omega_0}\frac{\partial\theta}{\partial\tau}, \tag{5b}$$

where we use the notation: $\xi = z/l$, $l = \sqrt{\alpha/(4\pi - \beta_0)}$ is the characteristic magnetic length, which is significantly less in terms of the problem than the film thickness $l \ll L$; $\omega_0 = g(4\pi - \beta_0)m_0$ is the characteristic frequency of the magnetic system; $h_0 = H_0/((4\pi - \beta_0)m_0)$ is the normalized constant component of the magnetic field; $h_1 = H_1/((4\pi - \beta_0)m_0)$ is the normalized variable component of the magnetic field; ω is the oscillation frequency of the variable component of the external magnetic field; $\tau = t\omega$ is the dimensionless time parameter of the problem, and $k = \frac{l}{L}\frac{\Delta\beta}{4\pi - \beta_0} \ll 1$ is the coefficient determining the gradient anisotropy.

The fact that the anisotropy gradient is small in magnitude will be used later in the approximate calculations.

The following conditions are applied to the variable component of the field: $\omega/\omega_0 \ll 1$, $h_1 \ll h_0$.

At the top and bottom surfaces of the film, without fixing the magnetization, the following boundary conditions must be satisfied:

$$\frac{\partial\theta}{\partial\xi} = 0\bigg|_{\xi=0,\frac{L}{l}}, \quad \sin\theta\frac{\partial\varphi}{\partial\xi} = 0\bigg|_{\xi=0,\frac{L}{l}}. \tag{6}$$

Note that the terms with small parameters (γ, ω/ω_0, h_1) are collected on the right-hand sides of Eqs. (5a) and (5b). Therefore, for finding their solution, we use the elements of perturbation theory.

We will seek the solution of Eqs. (5a) and (5b) in the form

$$\begin{aligned} \varphi(\xi - \xi_0, \tau) &= \Phi(\tau) + \delta\Phi(\xi - \xi_0, \tau), \\ \theta(\xi - \xi_0, \tau) &= \Theta(\xi - \xi_0) + \delta\Theta(\xi - \xi_0, \tau), \end{aligned} \tag{7}$$

where Φ, Θ is the main approximation that is determined by the equations

$$\frac{\partial}{\partial\xi}\sin^2\Theta\frac{\partial\Phi}{\partial\xi} = 0, \tag{8a}$$

$$-\frac{\partial^2\Theta}{\partial\xi^2} + \sin\Theta\left(h_0 - \left(1 - k\,\xi - \left(\frac{\partial\Phi}{\partial\xi}\right)^2\right)\cos\Theta\right) = 0. \tag{8b}$$

The parameter ξ_0 was introduced into the representation of the angular variables (7) and can be interpreted as a phase boundary with the different character of the magnetization.

It follows from Eq. (8a) that, in the main approximation, the angle Φ is an arbitrary function of time and does not depend on the coordinate ξ. Equation (8b) can be simplified as follows:

$$-\frac{\partial^2\Theta}{\partial\xi^2} + \sin\Theta(h_0 - (1 - k\,\xi)\cos\Theta) = 0. \tag{9}$$

Based on the type of Eq. (9), we can take, as the phase boundary ξ_0, the inflection point of the polar angle of the magnetization, at which the following condition holds: $\frac{\partial^2\Theta}{\partial\xi^2} = 0\big|_{\xi=\xi_0}$. On the both sides of this point, the polar angle of the magnetization tends to different limits. At the point on the phase boundary $\cos\Theta = h_0/(1 - k\xi_0)$, the derivative $\frac{\partial\Theta}{\partial\xi}$ has an extremum. Figure 4 qualitatively shows the dependence of the angle Θ and the auxiliary function

$$f(\xi) = \begin{cases} \arccos(h_0/(1 - k\,\xi)), & h_0/(1 - k\xi) < 1, \\ 0, & h_0/(1 - k\xi) \geqslant 1. \end{cases} \tag{10}$$

The derivatives $\partial\Theta/\partial\xi$ and $df/d\xi$ are also presented there.

Figure 4 is based on a qualitative analysis of Eq. (9), which yields $\Theta \approx f(\xi)$ with high accuracy. There is only one exception — a narrow region $|\xi - \xi_0| \sim 1$. Therefore, in the calculation, the angle value Θ will be replaced by a function $f(\xi)$ upon possibility. The point ξ_1 at which $h_0/(1 - k\xi_1) = 1$ and $f(\xi_1) = 0$ is a special. It coincides with the boundary phases of the magnetization without regard for the exchange interaction. It should be noted that the points ξ_0, ξ_1 are close $|\xi_1 - \xi_0| \sim 1$, so that they can be taken as identical ones during some approximate calculations.

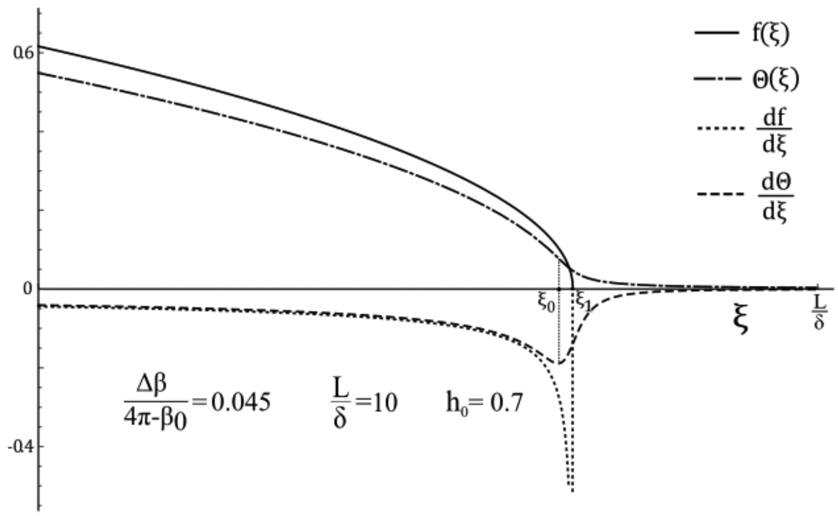

Fig. 4. Dependence of the deviation of the magnetization vector $\mathbf{\Theta}$ from film's normal on the coordinate ξ.

Note that the coexistence of the different orientation phases of the magnetization, as presented in Fig. 4, is possible in a relatively narrow field range:

$$0 < 1 - h_0 < \Delta\beta/(4\pi - \beta_0). \tag{11}$$

Under condition (11), the phase boundary ξ_1 will be located between the top and bottom surfaces of the film.

Under the influence of temporal or spatial perturbations, not only small corrections $\delta\Phi$ and $\delta\Theta$ to the magnetization angles will arise, but also the phase boundary coordinate, which is a function of time, will be changed. Therefore, we assume in what follows that the coordinate of the phase boundary has the form $\xi_0(\tau) = \mathrm{X} + u(\tau)$, where X is the stationary position of the interphase boundary, $u(\tau)$ is the correction to the coordinate of the phase boundary caused by perturbations. Thus, in constructing this theory, the coordinate $\xi_0(\tau)$ plays the role of a soft mode.

Expanding Eqs. (5a) and (5b) up to linear corrections $\delta\Phi$, $\delta\Theta$ and considering the displacement of phase boundaries by $u(\tau)$, we obtain

$$\frac{\partial}{\partial\eta}\sin^2\Theta\frac{\partial\delta\Phi}{\partial\eta} = \sin\Theta\left(\gamma\, h_0\,\sin\Phi + \frac{\omega}{\omega_0}\frac{\partial\Theta}{\partial\tau}\right) + \lambda_2\frac{\omega}{\omega_0}\sin^2\Theta\frac{\partial\Phi}{\partial\tau}, \tag{12a}$$

$$\left(-\frac{\partial^2}{\partial\eta^2} + h_0\cos\Theta - (1-k\,\eta)\cos 2\Theta\right)\delta\Theta$$
$$= -ku\,\sin\Theta\cos\Theta + \gamma h_0\cos\Phi\cos\Theta + \sin\Theta\left(\frac{\omega}{\omega_0}\frac{\partial\Phi}{\partial\tau} - h_1\cos\tau\right) - \lambda_1\frac{\omega}{\omega_0}\frac{\partial\Theta}{\partial\tau}, \tag{12b}$$

where the new variable $\eta = \xi - u(\tau)$ is introduced. In this case, it is assumed in Eqs. (12a) and (12b) that $\Theta = \Theta(\eta - \mathrm{X})$, and it is a solution of the equation $-\partial^2\Theta/\partial\eta^2 + \sin\Theta(h_0 - (1-k\,\eta)\cos\Theta) = 0$. It should be noted that the conditions for applying the method of successive approximations require the small value of $k|u| \ll 1$. Therefore, according to the accepted condition $k \ll 1$, the dynamic correction $u(\tau)$ to the interphase boundary coordinate takes an enough large value.

We proceed with constructing the system of equations for a "reduced description" of the magnetization dynamics in the variables $u(\tau)$, $\Phi(\tau)$, similar to the Slonczewski equations for the domain boundaries[14] or to the dynamical equations of the band domain.[15,16]

These equations can be obtained from the Fredholm alternative for the solvability of nonhomogeneous linear differential equations, which implies that the right-hand side of the linear equation must be orthogonal to the solution of the homogeneous equation.

The solution of the homogeneous equation (12a) $\delta\Phi = \text{const}$, and the homogeneous solution of Eq. (12b), neglecting small corrections proportional $k \ll 1$, coincides with the derivative of the spatial coordinate (this can be verified by taking the derivative of expression (9) with respect to the coordinate ξ).

Thus, the system of equations of the reduced description for the variables $u(\tau)$, $\Phi(\tau)$ is determined by the conditions

$$\begin{aligned}
&\int_0^{L/l} d\eta\, \sin\Theta\left(\gamma\, h_0\, \sin\Phi - \frac{\omega}{\omega_0}\frac{du}{d\tau}\frac{\partial\Theta}{\partial\eta} + \lambda_2\frac{\omega}{\omega_0}\sin\Theta\frac{d\Phi}{d\tau}\right) = 0,\\
&\int_0^{L/l} d\eta\, \frac{\partial\Theta}{\partial\eta}\left(\cos\Theta(\gamma h_0\cos\Phi - ku\,\sin\Theta)\right.\\
&\qquad\left. + \sin\Theta\left(\frac{\omega}{\omega_0}\frac{d\Phi}{d\tau} - h_1\cos\tau\right) + \lambda_1\frac{\omega}{\omega_0}\frac{du}{d\tau}\frac{\partial\Theta}{\partial\eta}\right) = 0.
\end{aligned} \tag{13}$$

For integrating (13) and processing the results, we will use the function $f(\eta)$, assuming $\Theta \approx f(\eta)$, instead of the exact value of the angle $\Theta(\eta - \xi_0)$. The results of the integration are elementary functions. For details of the calculations of individual coefficients of Eqs. (13), see Appendix A. To represent the results of calculations and to simplify their subsequent analysis, we introduce the notation $\varepsilon = 1 - h_0$ and, according to condition (11) assume that $\varepsilon < \Delta\beta/(4\pi - \beta_0) \ll 1$. This condition corresponds to the low-gradient nature of the magnetic anisotropy modulation.

Performing calculations in (13) and considering the notation above, we obtain the system of equations in explicit form:

$$\frac{\omega}{\omega_0}\frac{d\,ku}{d\tau} + 2\lambda_2\varepsilon\frac{\omega}{\omega_0}\frac{d\,\Phi}{d\tau} + \frac{2\gamma\sqrt{2\varepsilon}}{3}\sin\Phi = 0, \tag{14a}$$

$$\left(1 + \lambda_1\frac{\omega}{\omega_0}\frac{\ln(\varepsilon/k)}{2\varepsilon}\frac{d}{d\tau}\right)ku - \frac{\omega}{\omega_0}\frac{d\Phi}{d\tau} = \gamma\sqrt{\frac{2}{\varepsilon}} - h_1\cos(\tau). \tag{14b}$$

It is obvious that, in the absence of an alternating field at the $h_1 = 0$, the solution of Eqs. (14a), (14b) is of the type $ku_0 = \gamma\sqrt{2/\varepsilon}$, $\Phi_0 = 0$. At the same time, in the assumption $|\Phi| \ll 1$, the equation for dynamic corrections ku_1 can be represented as

$$\begin{aligned}
&\left\{\omega^2\frac{d^2}{d\tau^2} + 2\Gamma\omega\frac{d}{d\tau} + \Omega^2\right\}ku_1 = -(\Omega^2/k)h_1\cos(\tau),\\
&\Gamma = \omega_0\left(\lambda_2\varepsilon + \lambda_1\frac{\gamma}{3}\frac{\ln(\varepsilon/k)}{\sqrt{2\varepsilon}}\right); \quad \Omega = \omega_0\sqrt{\frac{2\gamma}{3}}(2\varepsilon)^{1/4}.
\end{aligned} \tag{15}$$

To simplify further calculations, the terms proportional to $\alpha_G\varepsilon$ were excluded from the right-hand side of Eq. (15) as small quantities of the second order.

It follows from (15) that the motion of the interphase boundary in a film with modulated anisotropy in a small alternating field has the form of forced oscillations of a harmonic oscillator:

$$\begin{aligned}
u_1 &= -\frac{h_1}{k}\frac{\Omega^2\,\cos(\tau - \psi)}{\sqrt{(\Omega^2 - \omega^2)^2 + 4\Gamma^2\omega^2}},\\
\operatorname{tg}\psi &= \frac{2\Gamma\omega}{\Omega^2 - \omega^2}.
\end{aligned} \tag{16}$$

The eigenfrequency of the oscillator $\Omega = g(4\pi - \beta_0)m_0\sqrt{\frac{2\gamma}{3}}(2(1 - \frac{H_0}{(4\pi-\beta_0)m_0}))^{1/4}$ depends on the magnitude and direction of the external magnetic field and can vary over a wide range.

Under resonance conditions, the oscillation of the interphase boundary has a large amplitude and is determined by the expression $u_r = -\frac{h_1\Omega\sin(\tau)}{2k\Gamma}$.

The value of the angular variable Φ can be obtained from Eq. (14a) with regard for (16):

$$\Phi \approx -\frac{3k}{2\gamma\sqrt{2\varepsilon}}\frac{\omega}{\omega_0}\frac{d\,u_1}{d\tau} = -2\frac{\omega\omega_0 k}{\Omega^2}\frac{d\,u_1}{d\tau}. \tag{17}$$

Based on the results of calculations, it is easy to determine the dynamics of the magnetic moment of the system.

Thus, it follows from expression (7) that the corrections to the angular variables caused by changes in the position of the interphase boundary have the form

$$\begin{aligned} \theta &\approx \Theta(\xi - \mathrm{X}) - (u_0 + u_1(\tau))\frac{d\Theta(\xi - \mathrm{X})}{d\xi}, \\ \varphi &\approx \Phi(\tau) = -2\frac{\omega\omega_0 k}{\Omega^2}\frac{d\,u_1}{d\tau}. \end{aligned} \tag{18}$$

On the basis of relations (2) and (18), the integration of the magnetization components over the volume of a ferromagnetic film leads to the following value for the dynamic corrections to the magnetic moment caused by the oscillations of the position of the interphase boundary $u_1(\tau)$:

$$\begin{gathered} \delta M_i = V\,\mathrm{Re}(\chi_{iz}\,H_1\exp(i(\omega t - \psi))), \quad i = x, y, z, \\ \begin{pmatrix} \chi_{xz} \\ \chi_{yz} \\ \chi_{zz} \end{pmatrix} = \frac{1}{\Delta\beta\sqrt{(1-\omega^2/\Omega^2)^2 + (2\Gamma\omega/\Omega^2)^2}}\begin{pmatrix} (2\varepsilon)^{1/2} \\ -i(2\varepsilon^{5/4}\omega/\Omega\sqrt{3\gamma}) \\ \varepsilon \end{pmatrix}, \end{gathered} \tag{19}$$

where V is the volume of the magnetic film, and the coefficients χ_{iz} are treated as components of the magnetic susceptibility tensor, which can be determined experimentally. The experimental setup in this work allowed us to distinguish the normal component of the magnetic susceptibility tensor χ_{zz}. The comparison of the experimental data and the results of theoretical calculations for the given parameters is presented in Fig. 5. At a qualitative level, there is a good correspondence.

Of special interest is the dependence of the susceptibility on the modulation of the anisotropy constant $\Delta\beta$, as, on the one hand, the susceptibility increases, and, on the other hand, the field interval for the existence of the phase boundary and, correspondingly, the described effects is reduced.

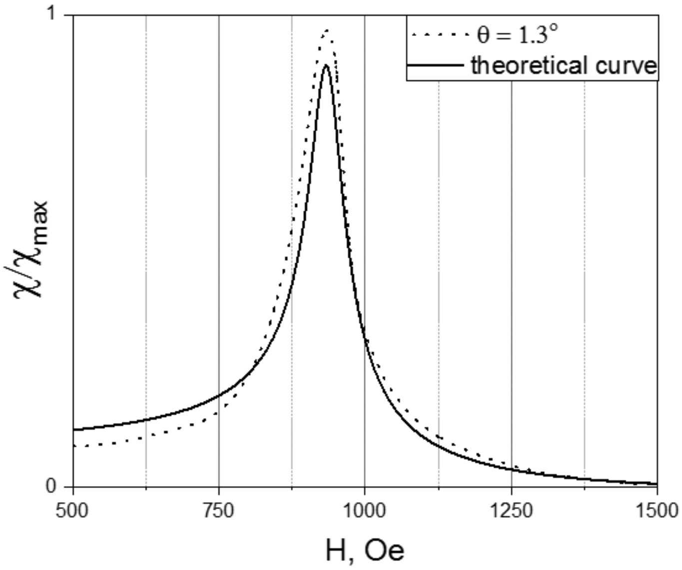

Fig. 5. Magnetic susceptibility $\boldsymbol{\chi}$ as a function of the magnetic field $\mathbf{H}_\theta$.

3.1. *Influence of magnetic field perturbations on the magnetic susceptibility of the system*

The following effect, which can have practical application, is associated with a sharp (root) dependence of the natural frequency Ω on the magnitude of the field $\varepsilon = 1 - h_0$ and the angle of the direction γ (15). This fact leads to a special sensitivity of the absorption characteristics to the indicated quantities in a vicinity of the resonance for small values of ε, $|\gamma| \ll 1$. This circumstance makes it possible to use this system as a functional element with high selectivity to the magnitude and direction of the magnetic field.

Indeed, when tuned to "resonance", the magnetic susceptibility of the film will react to the slightest external disturbances. Let us demonstrate this by the example of the magnetic susceptibility tensor component χ_{zz}, which can be easily determined experimentally. Suppose that the magnetic field sensor contains, as a functional element, the previously considered ferrite–garnet film and has adjustable magnetic field sources. By choosing a control field $\mathbf{h} = (\gamma h_0,\, 0,\, h_0)$, we satisfy the resonance condition so that the oscillation frequency of the interphase boundary coincides with the frequency of the alternating external field $\Omega = \omega_0\sqrt{\frac{2\gamma}{3}}\,(2\varepsilon)^{1/4}$. Under these conditions, the magnetic susceptibility component has a maximum value $(\chi_{zz})_{\max} = \frac{\Omega\varepsilon}{2\,\Delta\beta\,\Gamma}$. If, further, this sensor is introduced into the external magnetic field with components $\Delta\mathbf{h} = (\Delta h_x, 0, \Delta h_z)$, the resultant magnetic field acting on the film becomes equal to $\mathbf{h} = (\gamma h_0 + \Delta h_x,\, 0,\, h_0 + \Delta h_z)$, and the resonance condition is violated.

In this case, the ratio of the component of the magnetic susceptibility tensor to its maximum value will be determined by the relation

$$\frac{\chi_{zz}}{(\chi_{zz})_{\max}} = \frac{1}{\sqrt{1 + \frac{\Omega^2}{\Gamma^2}\left(\frac{\Delta h_x}{2\gamma} - \frac{\Delta h_z}{4\varepsilon}\right)^2}}. \tag{20}$$

Since the attenuation parameters of the magnetization oscillations for epitaxial ferrite–garnet films have a relatively small value ($\Gamma/\Omega \sim 10^{-2}$),[14] we can see that, even for weak perturbations of the external magnetic field $\Delta h_x/\gamma$, $\Delta h_z/\varepsilon \sim 10^{-1}$, the magnetic susceptibility of the system falls by an order of magnitude relative to its value at the resonance. These changes will be approximated by the relation

$$\frac{(\chi_{zz})_{\max}}{\chi_{zz}} \approx \frac{\Omega}{\Gamma}\left(\frac{\Delta h_x}{2\gamma} - \frac{\Delta h_z}{4\varepsilon}\right). \tag{21}$$

Obviously, for the magnetic field sensors formed on the basis of ferrite–garnet films with typical parameters $4\pi - \beta_0 \sim 1$, $m_0 \approx 20G$ for tuning characteristics $\gamma \approx 0.05$, $\varepsilon \approx 0.1$, the constant magnetic fields of magnitude $|\Delta\mathbf{H}| \sim 10^{-1} \div 10^{-2}$ will be available to the detection.

4. Conclusion

It follows from expression (21) that the various components of the magnetic field affect the magnetic susceptibility in the same way. Therefore, to determine the direction of the magnetic field $\Delta\mathbf{H}$, a selection should be carried out, for example, by changing the orientation of the functional element (the ferrite–garnet film) in space. This problem concerns the design features of the instrument and is beyond the scope of this study.

Thus, it is demonstrated that, under conditions of spatial inhomogeneity of the magnetic parameters in a ferromagnetic film, an interphase boundary separating regions with different magnetization properties can be formed. This boundary encompasses the entire plane of the film, has a high mobility typical of ordinary domain walls, and the resonance conditions of its oscillations are extremely sensitive to the parameters of the magnetic field. It is natural to use such a system as a highly sensitive, narrowly directed magnetic field detector.

Appendix A

A certain difficulty arises, when calculating the integral $\int_0^{L/l} d\eta(\partial\Theta/\partial\eta t)^2$, replacing $\Theta \to f(\eta)$. As follows from definition (10), the function $f(\eta)$ has nonzero values in the interval $0 \leqslant \eta < \eta_1$ (the point η_1 at which $f(\eta_1) = 0$, and the integration region during the transition $\Theta \to f(\eta)$ is limited by this interval. At the upper boundary of the interval — at a point, $(\partial f/\partial\eta)^2 \sim 1/(\eta - \eta_1)$, and the logarithmic uncertainty arises upon the integration at the upper limit. In order to avoid it in the approximate calculations, we set the upper limit of the integration to be $\eta_1 - a$, where a is a positive value of an arbitrary order $a \sim 1 \ll \eta_1$.

Then:

$$\int_0^{L/l} d\eta \left(\frac{\partial\Theta}{\partial\eta}\right)^2 \approx \int_0^{\eta_1-\alpha} d\eta \left(\frac{\partial f}{\partial\eta}\right)$$
$$= k^2 \int_0^{\eta_1-\alpha} d\eta \frac{h_0^2}{(1-k\eta)^2} \frac{1}{(1-k\eta)^2 - h_0^2}$$
$$= \frac{k}{2h_0}\left(\ln \frac{2h_0(1-h_0)}{k(1+h_0)} - \ln\alpha - \frac{2(1-h_0-k\alpha)}{h_0+k\alpha}\right). \quad \text{(A.1)}$$

Considering further that, according to the conditions of the problem, $1-h_0 = \varepsilon \ll 1$, we represent (A.1) in the form:

$$\int_0^{L/l} d\eta \left(\frac{\partial\Theta}{\partial\eta}\right)^2 \approx \frac{k}{2}\ln\left(\frac{\varepsilon}{k}\right). \quad \text{(A.2)}$$

In expression (A.2), we have neglected the small quantities $\ln(a)/\ln(2/k) \ll 1$, $\varepsilon/\ln(2/k) \ll 1$ and obtained an approximate value of the unknown quantity.

References

1. V. N. Dudorov, V. V. Randoshkin and R. V. Telesnin, *Sov. Phys. Usp.* **20**, 505 (1977).
2. S. Kruchinin, Yu. Dzezherya and J. Annett, *Supercond. Sci. Technol.* **19**, 381 (2006).
3. Yu. Dzezherya, I. Yu. Novak and S. Kruchinin, *Supercond. Sci. Technol.* **23**, 105011 (2010).
4. V. I. Butrim, S. V. Dubinko and Yu. N. Mitsay, *Fiz. Tverd. Tela Solid State Phys.* **45**, 1052 (2003).
5. V. G. Bar'yakhtar and Yu. I. Gorobets, *Tsilindricheskie Magnitnyie Domenyi i ikh Reshetki* (*Cylindrical Magnetic Domains and Their Lattices*) (Naukova dumka, Kiev, 1988) (in Russian).
6. V. V. Randoshkin *et al.*, *Z. Tek. Fiz. J. Tech. Phys.* **70**, 118 (2000).
7. A. Yu. Kozhuhar, *Dinamika elektronnykh sostoyaniy v sloistyih sistemah na osnove epitaksialnyih ferrit–granatovyih plenok* (*Dynamics of electronic states in layered systems based on epitaxial ferrite–garnet films*), Doctral Dissertation, Moscow (2005) (in Russian).
8. A. R. Prokopov *et al.*, *Fiz. Tverd. Tela* (*Solid State Phys.*) **39**, 1415 (1997).
9. V. G. Bar'yahtar, L. A. Suslin and A. O. Hrebtov, *Fiz. Tverd. Tela* (*Solid State Phys.*) **28**, 2246 (1986).
10. Yu. I. Gorobets, O. V. Ilchishin and V. F. Klepikov, *Fizika Metal. Metal.* (*Phys. Metals Metal.*) **50**, 261 (1980).
11. V. G. Bar'yakhtar *et al.*, *Sb. Nauchn. Trudov.* (*Const. Devices CMD*) **95**, 7 (1982).
12. A. O. Hrebtov and E. F. Hodosov, *Pisma ZhTF* (*Lett. J. Tech. Phys.*) **10**, 1127 (1984).
13. V. G. Bar'yakhtar, B. A. Ivanov and K. A. Safaryan, *Solid State Commun.* **72**, 1117 (1989).
14. A. P. Malozemoff, J. C. Slonczewski and R. Wolfe, *Magnetic Domain Walls in Bubble Materials* (Academic Press, New York, 1979).
15. Yu. I. Dzhezherya, *Zh. Eksp. Teor. Fiz.* (*J. Exp. Theor. Phys.*) **115**, 1315 (1999).
16. Yu. I. Dzhezherya, *Zh. Eksp. Teor. Fiz.* (*J. Exp. Theor. Phys.*) **116**, 1694 (1999).