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MODERN APPROACH TO THE CALCULATION OF THE CORRELATION FUNCTION IN SUPERCONDUCTIVITY MODELS

N. N. BOGOLUBOV, Jr.
V.A. Steklov Mathematical Institute, Moscow, Russia
nickolai@bogol.mian.su

E. N. BOGOLUBOVA
Moscow State University, Moscow, Russia

S. P. KRUCHININ
Bogolubov Institute for Theoretical Physics, Kiev, Ukraine
skruchin@i.com.ua

The superconductivity Hamiltonian model with attraction is investigated. An asymptotic relation for average calculation is constructed in the model and the approximating Hamiltonians are presented.

In our previous articles an approximation procedure based on the so-called "approximating Hamiltonian" has been developed together with the conditions where the obtained solution becomes asymptotically exact in the limiting process of statistical mechanics.

The aim of the present article is to study the problem of the asymptotically exact definition of many time correlation functions of any order as well as correlation functions corresponding to the Green functions for systems with attraction. We have formulated a method with more a profound analysis between correlation functions, free energies and Hamiltonian approximation. We notice that our methods[1] for a Hamiltonian model with negative four-fermion interaction have been constructed in general form for free energies.[2]

Here, we use our previous results concerning the estimation of free energies for model and approximation systems; and further with respect to the results in Refs. 1, 3 and 4 we establish their connection with correlation functions. The estimation which we get will be an estimation for quasi-averages (with the usual two limit techniques, the first $V \to \infty$, and then $r > 0$, $r \to \infty$). We notice that the approach being applied for the case of the positive four-fermion interaction [5] in the case of the model system with negative interaction is not valid because the first case is, in principle, distinguished from the second one. In the first case of the positiveness four-fermion interaction, the proof technique is essentially based on the fact that the maximum of the free energy is constructed by means of the approximating Hamiltonian.

In the second case, the negative four-fermion interaction, the free-energy, calculated for the approximating Hamiltonian $H_0(C)$, will provide a minimum relative to parameter C, essentially when using the $U - V$ Bogolubov transformation. The considered model problem is characterized by a Hamiltonian with a negative pair four-fermion interaction (so-called BCS Hamiltonian[6]):

$$\Gamma = T - 2V\varphi\varphi^\dagger g - r(\varphi + \varphi^\dagger)V\,, \tag{1}$$

where

$$\begin{aligned} T &= \sum_{(f)} T_f a_f^\dagger a_f\,, \quad T_f = \frac{p^2}{2m}\,, \quad g > 0\,, \\ \varphi &= \frac{1}{2V}\sum_{(f)} \lambda_f a_f^\dagger a_f^\dagger\,, \quad \varphi^\dagger = \frac{1}{2V}\sum_{(f)} \lambda_f a_f a_f\,, \end{aligned} \tag{2}$$

and V is the volume of the system, $f = (p, \sigma)$ is a set impulse p, and σ is the spin index possesing two values $\frac{1}{2}, -\frac{1}{2}$; p is an ordinary quasi-discrete momentum spectrum going into a continuous positive parameter which characterizes the intensity of interaction at $V \to \infty$.

Then we introduce terms $r(\varphi + \varphi^\dagger)V$ and $r > 0$ in the Hamiltonian source.

The functions λ_f and T_f satisfy the following additional conditions:

$$T_{-f} = T_f\,, \qquad \lambda_f = -\lambda_f\,, \tag{3}$$

$$\frac{1}{2V}\sum_{(f)} |\lambda_f| \le k_1\,, \quad \frac{1}{2V}\sum_{(f)} |T_f \lambda_f| \le k_2\,, \quad \frac{1}{V}\sum_{(f)} (\lambda_f)^2 > k_3\,, \tag{4}$$

where k_1, k_2, k_3 are constants at $V \to \infty$. The free-energy per unit volume corresponding to the system of non-interacting particles is finite.

We shall show that the correlation functions composed from the product of Fermi operators of any order for the model (1) can be calculated with asymptotic accuracy on the approximating Hamiltonian (8):

$$\Gamma^0 = T - 2VCg(\varphi^\dagger + \varphi) + 2VC^2 g - r(\varphi + \varphi^\dagger)V\,, \tag{5}$$

which represents a quadratic form of Fermi operators, and can be reduced to a diagonal form. The complex constants C in Eq. (5) must be found according to Refs. 7–11 from the minimum condition for free energy constructed from the Hamiltonian approximation (5):

$$f_{\Gamma^0}(C)\,, \qquad f_{\Gamma^0}(C) = \min\,. \qquad \text{and hence} \qquad \frac{\partial f_{\Gamma^0}}{\partial C} = 0\,.$$

To prove this difference we apply the method obtained in our previous works, and in so doing we obtain the inequality:

$$C \le f_{\Gamma^0} - f_\Gamma \le \frac{L}{V^{2/5}}\,, \qquad L = \text{const}\,.$$

This result has been used in numerous applications. For example, using this result, Hertel and Thirring calculated the free energy in the thermodynamic limit for a model describing a system of mutually attracting fermions.[12] A similar exactly solvable model of a crystal,

$$H = \sum_{(q)} T(q) a_q^\dagger a_q + \frac{V}{2} \sum_{(q)} \lambda(q) \rho_q \rho_{-q}, \qquad \rho_q = \frac{1}{V} \sum_{(k)} a_{k+q}^\dagger a_k$$

was considered by Bazarov.[13] The idea of the approximating Hamiltonian was developed in Refs. 20–23. Let us consider the correlation function averages composed from the product of an arbitrary number of Fermi operators based on the Hamiltonian model (1) and the corresponding approximation (5):

$$\langle \mathcal{H} \rangle_\Gamma = \frac{Sp\, e^{-\frac{\Gamma}{\theta}} \mathcal{H}}{Sp\, e^{-\frac{\Gamma}{\theta}}}\,, \tag{6}$$

$$\langle \mathcal{H} \rangle_{\Gamma_0} = \frac{Sp\, e^{-\frac{\Gamma_0}{\theta}} \mathcal{H}}{Sp\, e^{-\frac{\Gamma_0}{\theta}}}\,, \tag{7}$$

where $\mathcal{H}$ represents some product from Fermi operators with arbitrary number as follows

$$\mathcal{H} = \cdots a_{f_1}(f_1) \cdots a_{f_s}(f_s) \cdots .$$

These selection rules will be used to know in advance what averages $\langle \mathcal{H} \rangle_\Gamma$, $\langle \mathcal{H} \rangle_{\Gamma_0}$ with an arbitrary set of Fermi operators are equal to zero and so eliminate them from consideration. We notice that our model (1) and approximation system (5) are invariant with respect to the following special gradient transformation:

$$\begin{aligned} & a_{p\sigma} \to e^{i\Phi} a_{p\sigma}\,, \\ \text{for} \quad & p = p_0, -p_0\,, \\ \text{and} \quad & a_{p\sigma} \to a_{p\sigma}\,, \\ & p \neq p_0\,. \end{aligned}$$

For the sake of convenience we can write it in the form:

$$\begin{aligned} a_{f_0} &\to e^{i\Phi} a_{f_0}, \qquad a_{f_0}^\dagger \to e^{-i\Phi} a_{f_0}^\dagger\,, \\ a_{-f_0} &\to e^{-i\Phi} a_{-f_0}, \qquad a_{-f_0}^\dagger \to e^{i\Phi} a_{-f_0}^\dagger\,, \end{aligned} \tag{8}$$

(a_f and $a_f^\dagger$ are not valid if $f \neq f_0, -f_0$) keeping in mind that such a transformation is with impulse variable p. To obtain such a transformation (8) we introduce the unitary operator:

$$Z = e^{i\Phi(n_{p_0} - n_{-p_0})}, \qquad Z^\dagger = e^{-i\Phi(n_{p_0} - n_{-p_0})}\,,$$

$$ZZ^\dagger = 1\,,$$

where $n_{p_0} - n_{-p_0}$ is an operator of difference number particles with momentum p and $-p_0$ being the integral of motion for system (1). Using such an operator from the left and putting a_{-f}, a_f, $a_f^\dagger$, $a_{-f}^\dagger$ we have the gradient transformations:

$$a_{f_0} \to Z^\dagger a_{f_0} Z = e^{i\Phi} a_{f_0}, \qquad a_{-f_0} \to Z^\dagger a_{-f_0} Z = e^{-i\Phi} a_{-f_0} .$$

Returning back to the averages (6) and (7). Suppose that in such a product of Fermi operators, one can mark out the pair combinations in the form:

$$a_{f_0} a_{-f_0}, \ldots, a_{g_0}^\dagger a_{g_0}^\dagger, \ldots, a_{h_0} a_{-h_0}, \ldots, a_{s_0}^\dagger a_{s_0} . \tag{9}$$

Then, taking into account the gradient transformation (8) we see that the "phase" in such combinations becomes equal and therefore averages, (6) and (7) which contain such pair combinations of operators, in general, may not be equal to zero. In contrast, if there is at least one unpaired operator in the considered product of Fermi operators, then it is easy to see that such averages will be equal to zero. In particular, averages composed of an odd number of Fermi operators are equal to zero.

Now let us find an estimation of the difference

$$\langle \mathcal{H} \rangle_\Gamma - \langle \mathcal{H} \rangle_{\Gamma_0} . \tag{10}$$

To construct the corresponding estimates, it is convenient to transform inequalities from the old Fermi operators a_f, $a_f^\dagger$ to the new ones α_f, $\alpha_f^\dagger$, using the Bogoliubov transformation $U - V$ [10]:

$$\begin{aligned} a_f &= u_f \alpha_f - v_f \alpha_{-f}^\dagger , \\ a_f^\dagger &= u_f \alpha_f^\dagger - v_f \alpha_{-f} , \end{aligned} \tag{11}$$

where functions u_f and v_f satisfy the condition of symmetry:

$$u_{-f} = u_f, \qquad v_{-f} = -v_f, \qquad u_f^2 + v_f^2 = 1 .$$

We put for u_f and v_f:

$$u_{-f} = \frac{1}{\sqrt{2}} \sqrt{1 + \frac{T_f}{E_f}}, \qquad v_{-f} = -\frac{\mathcal{E}(f)}{\sqrt{2}} \sqrt{1 - \frac{T_f}{E_f}} ,$$

where

$$E_f = \sqrt{T_f^2 + \lambda_f^2 (2Cg + r)^2} .$$

Approximating Hamiltonians under the transformation may be reduced to the diagonal form in the new Fermi operators:

$$\Gamma_0 = \sum_{(f)} E_f \alpha_f^\dagger \alpha_f + V \left\{ 2C^2 g - \frac{1}{2V} \sum_{(f)} (E_f - T_f) \right\} . \tag{12}$$

It is easy to notice that the averages $\langle \mathcal{H} \rangle_{\Gamma_0}$, constructed on the basis of such a model of the Hamiltonian, can be calculated using the Wick and Bloch methods.

Let us write down the motion equation for the model system (1) in the new Fermi operators keeping in mind their previous form:

$$i\frac{da_f}{dt} = T_f a_f - \lambda_f a_f^\dagger (2\varphi^\dagger g + r)\,,$$
$$i\frac{da_f^\dagger}{dt} = -T_f a_f^\dagger + \lambda_f (2\varphi g + r) a_{-f}\,. \tag{13}$$

Taking into account (11) we introduce new Fermi operators:

$$\alpha_f = a_f u_f + a_f^\dagger v_f\,,$$
$$\alpha_f^\dagger = a_f^\dagger u_f + a_{-f} v_f\,. \tag{14}$$

By differentiating (14) with respect to t and the expressed derivatives $\frac{da_f}{dt}$, $\frac{da_f^\dagger}{dt}$ through the right-hand side of motion equation (13) we obtain

$$\begin{aligned} i\frac{d\alpha_f^\dagger}{dt} &= iu_f\frac{da_f^\dagger}{dt} + iv_f\frac{da_{-f}}{dt} \\ &= u_f\{-T_f a_f^\dagger + \lambda_f(2\varphi g + r)a_{-f}\} + v_f\{T_f a_{-f} + \lambda_f a_f^\dagger(2\varphi^\dagger g + r)\} \\ &= -\,a_f^\dagger\{T_f u_f - \lambda_f v_f(2\varphi^\dagger g + r)\} + \{u_f\lambda_f(2\varphi g + r) + v_f T_f\}a_{-f} \\ &= -\,a_f^\dagger\{T_f u_f - \lambda_f v_f(2Cg + r)\} + \{u_f\lambda_f(2Cg + r) + v_f T_f\}a_{-f} \\ &\quad + u_f\lambda_f(2\varphi g - 2Cg)a_{-f} + v_f\lambda_f a_f^\dagger(2\varphi^\dagger g - 2Cg)\,. \end{aligned}$$

Taking into account the identity equations,

$$T_f u_f - \lambda_f v_f(2Cg + r) = \sqrt{(2Cg + r)^2\lambda_f^2 + T_f^2 u_f^2}\,,$$
$$T_f v_f + \lambda_f u_f(2Cg + r) = \sqrt{(2Cg + r)^2\lambda_f^2 + T_f^2 v_f^2}\,,$$

we find the motion equation in new Fermi operators:

$$i\frac{d\alpha_f^\dagger}{dt} + E_f\alpha_f^\dagger = R_f\,,$$
$$i\frac{d\alpha_f}{dt} - E_f\alpha_f = -R_f^\dagger\,, \tag{15}$$

where

$$R_f = R_f^{(1)} + R_f^{(2)}\,, \tag{16}$$

$$R_f^{(1)} = u_f\lambda_f(2\varphi g - 2Cg)a_{-f}, \qquad R_f^{(2)} = v_f\lambda_f a_f^\dagger(2\varphi^\dagger g - 2Cg)\,, \tag{17}$$

$$E_f = \sqrt{(2Cg + r)^2\lambda_f^2 + T_f^2}\,. \tag{18}$$

Let us consider now the correlation function:

$$\langle \alpha_f(t)\cdot \mathcal{B}(0)\rangle\,, \tag{19}$$

where $\mathcal{B}(0)$ represents some product from s Fermi operators. Let us write down the equation for averages by differentiating Eq. (19) with respect to t, and using the motion equation:

$$i\frac{d}{dt}\langle\alpha_f(t)\cdot\mathcal{B}(0)\rangle_\Gamma = E_F\langle\alpha_f(t)\cdot\mathcal{B}(0)\rangle_\Gamma - \langle R_f^\dagger(t)\cdot\mathcal{B}(0)\rangle_\Gamma\,, \tag{20}$$

$$i\frac{d}{dt}\langle\alpha_f^\dagger(t)\cdot\mathcal{B}(0)\rangle_\Gamma = -E_F\langle\alpha_f^\dagger(t)\cdot\mathcal{B}(0)\rangle_\Gamma - \langle R_f(t)\cdot\mathcal{B}(0)\rangle_\Gamma\,. \tag{21}$$

The solution of this equation is given by

$$\langle\alpha_f(t)\cdot\mathcal{B}(0)\rangle_\Gamma = \langle\alpha_f(0)\cdot\mathcal{B}(0)\rangle_\Gamma e^{-iE_f t} + ie^{-iE_f t}\left\langle\int_0^t e^{iE_f t}R(t')\cdot\mathcal{B}(0)dt'\right\rangle_\Gamma\,,$$

$$\langle\alpha_f^\dagger(t)\cdot\mathcal{B}(0)\rangle_\Gamma = \left\langle\alpha_f^\dagger(0)\cdot\mathcal{B}(0)\right\rangle_\Gamma e^{iE_f t} - ie^{iE_f t}\left\langle\int_0^t e^{-iE_f t}R(t')\cdot\mathcal{B}(0)dt'\right\rangle_\Gamma\,,$$

from which we get

$$\left|\langle\alpha_f(t)\cdot\mathcal{B}(0)\rangle_\Gamma - \langle\alpha_f(0)\cdot\mathcal{B}(0)\rangle_\Gamma e^{-iE_f t}\right| \le \left|\left\langle\int_0^t e^{iE_f t}R(t')\cdot\mathcal{B}(0)dt'\right\rangle_\Gamma\right|\,, \tag{22}$$

$$\left|\langle\alpha_f^\dagger(t)\cdot\mathcal{B}(0)\rangle_\Gamma - \langle\alpha_f^\dagger(0)\cdot\mathcal{B}(0)\rangle_\Gamma e^{iE_f t}\right| \le \left|\left\langle\int_0^t e^{-iE_f t}R(t')\cdot\mathcal{B}(0)dt'\right\rangle_\Gamma\right|\,. \tag{23}$$

To construct the following estimation we use the inequality:

$$|\langle Y\cdot W\rangle| < \left\{\left|\langle YY^\dagger\rangle\right|\cdot\left|\langle W^\dagger W\rangle\right|\right\}^{\frac{1}{2}}\,. \tag{24}$$

The proof of this inequality and its spectral analog is given in Refs. 14–19. Making estimations of the right-hand side of Eqs. (22) and (23) and using inequalities (24), we obtain

$$\begin{aligned}\left|\left\langle\int_0^t e^{iE_f t}R_f^\dagger(t')\cdot\mathcal{B}(0)dt'\right\rangle_\Gamma\right| &\le \int_0^t \left|\langle e^{iE_f t}\cdot R_f^\dagger(t')\cdot\mathcal{B}(0)\rangle_\Gamma\right|dt' \\ &\le \int_0^t \left\{\left|\langle R_f^\dagger(t')\cdot R_f(t')\rangle_\Gamma\right|\cdot\left|\left\langle\mathcal{B}^\dagger(0)\cdot\mathcal{B}(0)\right\rangle_\Gamma\right|\right\}^{\frac{1}{2}}dt' \\ &\le |t|\left\{\left|\langle R_f^\dagger(0)\cdot R_f(0)\rangle_\Gamma\right|\cdot\left|\langle\mathcal{B}^\dagger(0)\cdot\mathcal{B}(0)\rangle_\Gamma\right|\right\}^{\frac{1}{2}}\,.\end{aligned} \tag{25}$$

By analogy way we get estimates:

$$\begin{aligned}\left|\left\langle\int_0^t e^{-iE_f t}R_f(t')\cdot\mathcal{B}(0)dt'\right\rangle_\Gamma\right| &\le \int_0^t \left|\langle e^{iE_f t}\cdot R_f^\dagger(t')\cdot\mathcal{B}(0)\rangle_\Gamma\right|dt' \\ &\le |t|\left\{\left|\langle R_f(0)\cdot R_f^\dagger(0)\rangle_\Gamma\right|\cdot\left|\langle\mathcal{B}^\dagger(0)\cdot\mathcal{B}(0)\rangle_\Gamma\right|\right\}^{\frac{1}{2}}\,.\end{aligned} \tag{26}$$

Let us find now estimates for the correlation function

$$\langle R_f^\dagger(0)\cdot R_f(0)\rangle_\Gamma, \qquad \langle R_f(0)\cdot R_f^\dagger(0)\rangle_\Gamma \tag{27}$$

in the right-hand side of inequalities (25) and (26). Using formulas (16)–(18) we write

$$\begin{aligned}\left|\langle R_f^\dagger(0)\cdot R_f(0)\rangle_\Gamma\right| &\le 2\langle R_f^{\dagger(1)}(0)\cdot R_f^{(1)}(0)\rangle_\Gamma + 2\langle R_f^{\dagger(2)}(0)\cdot R_f^{(2)}(0)\rangle_\Gamma\\ &\le 8g^2\lambda_f^2(u_f^2\langle a_{-f}^\dagger(\varphi^\dagger - C)(\varphi - C)a_{-f}\rangle_\Gamma\\ &\quad + v_f^2\langle(\varphi - C)a_f a_f^\dagger(\varphi^\dagger - C)\rangle_\Gamma)\,.\end{aligned}$$

Notice that $a_f\cdot a_f^\dagger$ is a positive operator with norm bounded by unity, so we find

$$\langle(\varphi - C)a_f a_f^\dagger(\varphi^\dagger - C)\rangle_\Gamma \le \langle(\varphi - C)(\varphi^\dagger - C)\rangle_\Gamma\,.$$

Let us estimate now the correlation averages:

$$\langle a_{-f}^\dagger(\varphi^\dagger - C)(\varphi - C)a_{-f}\rangle_\Gamma.$$

Keeping in mind that

$$\begin{aligned}(\varphi^\dagger - C)(\varphi - C) &= (\varphi - C)(\varphi^\dagger - C) + \varphi^\dagger\varphi - \varphi\varphi^\dagger\\ &\le \left|\varphi^\dagger\varphi - \varphi\varphi^\dagger\right| + \left|(\varphi - C)(\varphi^\dagger - C)\right|\,,\quad \left|\varphi^\dagger\varphi - \varphi\varphi^\dagger\right| \le \frac{k_3}{2V}\,,\end{aligned}$$

we obtain

$$\begin{aligned}\langle a_{-f}^\dagger(\varphi^\dagger - C)(\varphi - C)a_{-f}\rangle_\Gamma &\le \frac{k_3}{2V} + \langle(\varphi - C)a_{-f}^\dagger a_{-f}(\varphi^\dagger - C)\rangle_\Gamma\\ &\le \frac{k_3}{2V} + \langle(\varphi - C)(\varphi^\dagger - C)\rangle_\Gamma\,.\end{aligned}$$

Thus,

$$\langle R_f^\dagger(0)\cdot R_f(0)\rangle_\Gamma \le 8g^2\lambda_f^2\left(\langle(\varphi - C)(\varphi^\dagger - C)\rangle_\Gamma + \frac{u_f^2 k_3}{2V}\right)\,. \tag{28}$$

With estimations:

$$\begin{aligned}\langle R_f(0)\cdot R_f^\dagger(0)\rangle_\Gamma &\le 2\langle R_f^{(1)}(0)\cdot R_f^{\dagger(1)}(0)\rangle_\Gamma + 2\langle R_f^{(2)}(0)\cdot R_f^{\dagger(2)}(0)\rangle_\Gamma\\ &\le 8g^2\lambda_f^2(u_f^2\langle(\varphi - C)a_{-f}a_{-f}^\dagger(\varphi^\dagger - C)\rangle_\Gamma\\ &\quad + v_f^2 a_f^\dagger(\varphi^\dagger - C)(\varphi - C)a_f\rangle_\Gamma)\,,\end{aligned}$$

$$\langle(\varphi - C)a_{-f}a_{-f}^\dagger(\varphi^\dagger - C)\rangle_\Gamma \le \langle(\varphi - C)(\varphi^\dagger - C)\rangle_\Gamma\,,$$

$$\begin{aligned}\langle a_f^\dagger(\varphi^\dagger - C)(\varphi - C)a_f\rangle_\Gamma &\le \langle a_f^\dagger(\varphi - C)(\varphi^\dagger - C)a_f\rangle_\Gamma + \frac{k_3}{2V}\\ &\le \langle(\varphi - C)a_f^\dagger a_f(\varphi^\dagger - C)\rangle_\Gamma + \frac{k_3}{2V}\,,\end{aligned}$$

we get the following inequality:

$$\langle R_f(0) \cdot R_f^\dagger(0)\rangle_\Gamma \leq 8g^2\lambda_f^2(\langle\left(\varphi - C)(\varphi^\dagger - C)\rangle_\Gamma + \frac{v_f^2 k_3}{2V}\right) . \qquad (29)$$

Finally, the inequalities (28) and (29) can be rewritten in the form:

$$\langle R_f^\dagger(0) \cdot R_f(0)\rangle_\Gamma \leq 8g^2\lambda_f^2 \left\{\langle(\varphi - C)(\varphi^\dagger - C)\rangle_\Gamma + \frac{k_3}{2V}\right\} , \qquad (30)$$

$$\langle R_f(0) \cdot R_f^\dagger(0)\rangle_\Gamma \leq 8g^2\lambda_f^2 \left\{\langle(\varphi - C)(\varphi^\dagger - C)\rangle_\Gamma + \frac{k_3}{2V}\right\} . \qquad (31)$$

Notice that we can estimate the correlation averages $\langle(\varphi - C)(\varphi^\dagger - C)\rangle_\Gamma$ with Eq. (13), using the asymptotic closeness free energies, and further with respect to the lemma in Ref. 3 prove that

$$\langle(\varphi - C)(\varphi^\dagger - C)\rangle_\Gamma \leq \mathcal{E}_0\left(\frac{1}{V}, \delta\right) ,$$

$$\langle(\varphi^\dagger - C)(\varphi - C)\rangle_\Gamma \leq \mathcal{E}_0\left(\frac{1}{V}, \delta\right) ,$$

where $\mathcal{E}_0(\frac{1}{V}, \delta)$ is the expression for $V \to \infty$, and any positive fixed δ tends to zero.

Then, the estimations (30) and (31) can be rewritten as

$$\langle R_f^\dagger(0) \cdot R_f(0)\rangle_\Gamma \leq \mathcal{E}\left(\frac{1}{V}, \delta\right) , \qquad (32)$$

$$\langle R_f(0) \cdot R_f^\dagger(0)\rangle_\Gamma \leq \mathcal{E}\left(\frac{1}{V}, \delta\right) , \qquad (33)$$

where

$$\mathcal{E}\left(\frac{1}{V}, \delta\right) = 8g^2\lambda_f^2\left\{\mathcal{E}_0\left(\frac{1}{V}, \delta\right) + \frac{k_3}{2V}\right\} \to 0$$

for $V \to \infty$ and any fixed $\delta > 0$. Then, after making these transformations, we take into account that

$$\langle\mathcal{B}^\dagger(0)\mathcal{B}(0)\rangle \leq 1 ,$$

$$\left|\langle a_f(t) \cdot \mathcal{B}(0)\rangle_\Gamma - \langle\tilde{a}_f(t) \cdot \mathcal{B}(0)\rangle_\Gamma\right| \leq |t|\,\mathcal{E}\left(\frac{1}{V}, \delta\right) , \qquad (34)$$

$$\left|\langle a_f^\dagger(t) \cdot \mathcal{B}(0)\rangle_\Gamma - \langle\tilde{a}_f^\dagger(t) \cdot \mathcal{B}(0)\rangle_\Gamma\right| \leq |t|\,\mathcal{E}\left(\frac{1}{V}, \delta\right) , \qquad (35)$$

$$\left|\langle\mathcal{B}(0) \cdot a_f^\dagger(t)\rangle_\Gamma - \langle\mathcal{B}(0) \cdot \tilde{a}_f^\dagger(t)\rangle_\Gamma\right| \leq |t|\,\mathcal{E}\left(\frac{1}{V}, \delta\right) , \qquad (36)$$

where operators $\tilde{a}_f(t) = a_0 e^{-iE_f t}$, $\tilde{a}_f^\dagger(t) = a_0^\dagger e^{iE_f t}$ satisfy the motion equation with approximating Hamiltonian (5). From inequalities (34)–(36) we construct major estimations, which represent the asymptotic closeness correlation functions

$$\langle a_f^\dagger(0)\mathcal{B}(0)\rangle_\Gamma \simeq \langle a_f^\dagger(0)\mathcal{B}(0)\rangle_{\Gamma^0} \qquad \text{as} \quad V \to \infty \qquad \text{and} \quad \delta > 0\,.$$

Let us write down the spectral representation[14–19] as

$$\langle a_f^\dagger(t)\cdot\mathcal{B}(0)\rangle = \int_{-\infty}^{+\infty} \mathcal{J}_{a^\dagger\mathcal{B}}(\omega)e^{i\omega t}d\omega\,, \tag{37}$$

$$\langle\mathcal{B}(0)\cdot a_f^\dagger(t)\rangle = \int_{-\infty}^{+\infty} \mathcal{J}_{a^\dagger\mathcal{B}}(\omega)e^{\frac{\omega}{\theta}}\cdot e^{i\omega t}d\omega\,. \tag{38}$$

Then we find a function:

$$h_p(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left\{\frac{1}{\rho}F\left(\frac{\omega - E_f}{\rho}\right)\right\}e^{-i\omega t}d\omega\,, \tag{39}$$

$$\frac{1}{\rho}F\left(\frac{\omega - E_f}{\rho}\right) = \int_{-\infty}^{+\infty} h_\rho(t)e^{i\omega t}dt\,, \tag{40}$$

where

$$\begin{aligned} F(z) &\equiv 0 && \text{for} \quad |z| \geq 1\,, \\ F(z) &= (1-z^2)^2 && \text{for} \quad |z| < 1\,. \end{aligned} \tag{41}$$

Multiplying inequalities (35) and (36) by a function $h_\rho(t)$, integrating on t ($-\infty < t < +\infty$) and taking into account the spectral representations (37) and (38) we get

$$\left|\int_{-\infty}^{+\infty}\mathcal{J}_{a^\dagger\mathcal{B}}(\omega)F\left(\frac{\omega - E_f}{\rho}\right)d\omega - \langle a_f^\dagger(0)\cdot\mathcal{B}(0)\rangle_\Gamma\right| \leq \frac{\mathcal{E}_2\left(\frac{1}{V},\delta\right)}{\rho^2}\,, \tag{42}$$

$$\left|\int_{-\infty}^{+\infty}\mathcal{J}_{a^\dagger\mathcal{B}}(\omega)F\left(\frac{\omega - E_f}{\rho}\right)e^{\frac{\omega}{\theta}}d\omega - \langle\mathcal{B}(0)\cdot a_f^\dagger(0)\rangle_\Gamma\right| \leq \frac{\mathcal{E}_2\left(\frac{1}{V},\delta\right)}{\rho^2}\,, \tag{43}$$

where $\mathcal{E}_2(\frac{1}{V},\delta)$ tends to zero as $V \to \infty$ and for any fixed positive $\delta > 0$. Multiplying the inequality (43) by a factor $e^{-\frac{E_f}{\theta}}$ and subtracting the inequality (42), we get

$$\begin{aligned} &\left|\langle a_f^\dagger(0)\cdot\mathcal{B}(0)\rangle_\Gamma - \langle\mathcal{B}(0)\cdot a_f^\dagger(0)\rangle_\Gamma e^{-\frac{E_f}{\theta}}\right| \\ &\leq \mathcal{E}_2\left(\frac{1}{V_1},\delta\right)\left(1 + e^{-\frac{E_f}{\theta}}\right)\frac{1}{\rho^2} + \left|\int_{-\infty}^{+\infty}\mathcal{J}_{a^\dagger\mathcal{B}}(\omega)F\left(\frac{\omega - E_f}{\rho}\right)\left\{e^{\frac{\omega - E_f}{\theta}} - 1\right\}d\omega\right|\,. \end{aligned} \tag{44}$$

Let us estimate the second term of Eq. (44):

$$\begin{aligned}&\left|\int_{-\infty}^{+\infty}\mathcal{J}_{a^\dagger\mathcal{B}}(\omega)F\left(\frac{\omega-E_f}{\rho}\right)\left\{e^{\frac{\omega-E_f}{\theta}}-1\right\}d\omega\right|\\&\quad\le\int_{-\infty}^{+\infty}|\mathcal{J}_{a^\dagger\mathcal{B}}(\omega)|\cdot\left|F\left(\frac{\omega-E_f}{\rho}\right)\right|\cdot\left|e^{\frac{\omega-E_f}{\theta}}-1\right|d\omega\\&\quad\le\left|e^{\frac{\rho}{\theta}}-1\right|\cdot\left(\int_{-\infty}^{+\infty}\mathcal{J}_{aa^\dagger}(\omega)d\omega\right)^{\frac{1}{2}}\cdot\left(\int_{-\infty}^{+\infty}\mathcal{J}_{\mathcal{B}\mathcal{B}^\dagger}(\omega)d\omega\right)^{\frac{1}{2}}\end{aligned}\tag{45}$$

taking into account definition (41). Notice that

$$\int_{-\infty}^{+\infty}\mathcal{J}_{aa^\dagger}(\omega)d\omega\le\left|\langle a(0)\cdot a^\dagger(0)\rangle_\Gamma\right|\le 1\,,\tag{46}$$

$$\int_{-\infty}^{+\infty}\mathcal{J}_{\mathcal{B}\mathcal{B}^\dagger}(\omega)d\omega\le\left|\langle\mathcal{B}^\dagger(0)\cdot\mathcal{B}(0)\rangle_\Gamma\right|\le 1\,.\tag{47}$$

Taking into consideration the inequalities (45)–(47) we rewrite Eq. (44) as

$$\begin{aligned}&\left|\langle a_f^\dagger(0)\cdot\mathcal{B}(0)\rangle_\Gamma-\langle\mathcal{B}(0)\cdot a_f^\dagger(0)\rangle_\Gamma\cdot e^{-\frac{E_f}{\theta}}\right|\\&\qquad\le\mathcal{E}_2\left(\frac{1}{V},\delta\right)\cdot\frac{1}{\rho^2}\left(1+e^{\frac{E_f}{\theta}}\right)+\left|e^{\frac{\rho}{\theta}}-1\right|.\end{aligned}\tag{48}$$

ρ is a positive arbitrary value in all the above inequalities. The inequality (48) will hold for any positive ρ. To show the asymptotic smallness for some ρ-function from $\mathcal{E}_2(\frac{1}{V},\delta)$ for $V\to\infty$ and for any positive value $\delta>0$, $\rho\to 0$ and, on the other hand, strengthen inequality (48), we find for the ρ-function:

$$\rho=\left\{\mathcal{E}_2\left(\frac{1}{V},\delta\right)\right\}^{\frac{1}{8}}.$$

Rewriting the inequality (48) with some identity transformations:

$$\left|\langle a_f^\dagger\mathcal{B}\rangle_\Gamma+\langle a_f^\dagger\mathcal{B}\rangle_\Gamma e^{-\frac{E_f}{\theta}}-\langle a_f^\dagger\mathcal{B}+\mathcal{B}a_f^\dagger\rangle_\Gamma e^{-\frac{E_f}{\theta}}\right|\le\mathcal{E}_2^{\frac{3}{4}}\left(\frac{1}{V},\delta\right)\left(1+e^{-\frac{E_f}{\theta}}\right)+\left|e^{\frac{\varepsilon_2{}^{\frac{1}{8}}}{\theta}}-1\right|$$

or

$$\left|\langle a_f^\dagger\mathcal{B}\rangle_\Gamma-\frac{e^{-\frac{E_f}{\theta}}}{1+e^{-\frac{E_f}{\theta}}}\cdot\langle a_f^\dagger\mathcal{B}+\mathcal{B}a_f^\dagger\rangle_\Gamma\right|\le\bar{\mathcal{E}}_2\left(\frac{1}{V},\delta\right),\tag{49}$$

where

$$\bar{\mathcal{E}}_2\left(\frac{1}{V},\delta\right)=\mathcal{E}_2^{\frac{3}{4}}\left(\frac{1}{V},\delta\right)+\frac{\left|e^{\frac{\varepsilon_2{}^{\frac{1}{8}}}{\theta}}-1\right|}{1+e^{-\frac{E_f}{\theta}}}.$$

and applying the induction method we can prove for any n that

$$|\Delta_n| = |\langle\mathcal{H}\rangle_\Gamma - \langle\mathcal{H}\rangle_{\Gamma_0}| \leq \Pi \cdot \bar{\mathcal{E}}_2\left(\frac{1}{V},\delta\right). \tag{50}$$

Here $\mathcal{H}$ designates the product of Fermi operators of order n, $\Pi = \text{const.}$, $\mathcal{E}_2(\frac{1}{V},\delta) \to 0$ as $V \to \infty$ and for $\delta > 0$. According to our designation, $\mathcal{H}$ may be presented as $\mathcal{H} = a_f^\dagger\mathcal{B}$, $\mathcal{H} = a_f\mathcal{B}$, $\mathcal{H} = \mathcal{B}a_f$, $\mathcal{H} = \mathcal{B}a_f^\dagger$, where $\mathcal{B}$ presents the product operators $(n-1)$.

It is sufficient to consider one of these cases by analogous reasoning. Let us consider, for instance, the case if $\mathcal{H} = a_f^\dagger\mathcal{B}$. It follows from Eq. (49) that if $\langle a_f^\dagger\mathcal{B}\rangle$ consists of n operators, then $\langle a_f^\dagger\mathcal{B}+\mathcal{B}a_f^\dagger\rangle$ consists of $(n-2)$ operators. Here numbers $n \geq 2$, n are even. Notice, that we consider only the averages with even number n of Fermi operators, because averages constructed from an odd number of Fermi operators are equal to zero due to selection rules. Putting in Eq. (49) $\mathcal{B} = a_f$, we have

$$\left|\langle a_f^\dagger a_f\rangle_\Gamma - \frac{1}{1+e^{-\frac{E_f}{\theta}}}\right| \leq \bar{\mathcal{E}}_2\left(\frac{1}{V},\delta\right),$$

but if

$$\langle a_f^\dagger a_f\rangle_{\Gamma_0} = \frac{1}{(1+e^{\frac{E_f}{\theta}})}$$

then

$$\left|\langle a_f^\dagger a_f\rangle_\Gamma - \langle a_f^\dagger a_f\rangle_{\Gamma_0}\right| \leq \bar{\mathcal{E}}_2\left(\frac{1}{V},\delta\right).$$

By analogy we obtain for $\langle a_f a_f^\dagger\rangle$:

$$\left|\langle a_f a_f^\dagger\rangle_\Gamma - \langle a_f a_f^\dagger\rangle_{\Gamma_0}\right| \leq \bar{\mathcal{E}}_2\left(\frac{1}{V},\delta\right), \qquad \text{where} \qquad \langle a_f a_f^\dagger\rangle_{\Gamma_0} = \frac{e^{\frac{E_f}{\theta}}}{(1+e^{\frac{E_f}{\theta}})}.$$

As we can see, the inequality (49) holds for $n = 2$. It is possible to verify it for $n = 4$, $n = 6, \ldots$ and so on. To prove this inequality for any n suppose that

$$\left|\langle a_f^\dagger\mathcal{B}\rangle_\Gamma - \frac{e^{-\frac{E_f}{\theta}}}{1+e^{-\frac{E_f}{\theta}}}\langle a_f^\dagger\mathcal{B}+\mathcal{B}a_f^\dagger\rangle_{\Gamma_0}\right|_{\Pi_s=\text{const}} \leq \Pi_s \cdot \bar{\mathcal{E}}_2\left(\frac{1}{V},\delta\right).$$

Taking into account that

$$\langle a_f^\dagger\mathcal{B}\rangle_{\Gamma_0} = e^{-\frac{E_f}{\theta}}\langle\mathcal{B}a_f^\dagger\rangle_{\Gamma_0}$$

we have

$$|\Delta_s| = \left|\langle a_f^\dagger\mathcal{B}\rangle_\Gamma - \langle a_f^\dagger\mathcal{B}\rangle_{\Gamma_0}\right| \leq \Pi_s \cdot \bar{\mathcal{E}}_2\left(\frac{1}{V},\delta\right), \tag{51}$$

where the right-hand side for $V \to \infty$ and for any $\delta > 0$ tends to zero. Further, on the basis of this proposition we prove the truthfulness of Eq. (51) for $n = s + 2$:

$$|\Delta_{s+2}| \leq \Pi_{s+2} \cdot \bar{\mathcal{E}}_2 \left(\frac{1}{V}, \delta \right) .$$

So, from the true inequality (51) this is true for $n = s$ and for $n = s + 2$. And for $n = 2$ this inequality holds true, consequently the same holds for $n = 4$, $n = 6$, and so on. By successively adding a pair of operators it is possible to construct any even number of operators, so the inequality assertion (50) holds for any even number n. With arbitrariness $\mathcal{H}$ we can apply inequality (50) to any possible set of Fermi operators.

Remember that here we do not consider any arbitrary combination of Fermi operators but only that which does not convert to zero averages:

$$\langle \mathcal{H} \rangle_\Gamma, \langle \mathcal{H} \rangle_{\Gamma_0} .$$

It is convenient to consider the normal product of operators when all creation operators stay on the left of the annihilation operators. It is clear that any arbitrary order of following operators can always be reduced to the normal form. Let us take into consideration that "old" Fermi operators connect with "new" ones with respect to the canonical transformation $\mathcal{U}_f - \mathcal{V}_f$ (Bololiubov transformations) with bounded coefficients $\mathcal{U}_f, \mathcal{V}_f$.

Thus we have

$$|\langle \mathcal{H} \rangle_\Gamma - \langle \mathcal{H} \rangle_{\Gamma_0}| \leq \Pi' \cdot \bar{\mathcal{E}}_2 \left(\frac{1}{V}, \delta \right), \quad \Pi' = \text{const.},$$

where $\bar{\mathcal{E}}_2(\frac{1}{V}, \delta)$ for $V \to \infty$ and for positive number δ tending to zero. Here we consider averages without time dependence. However, for two time dependences or many time dependences the analogous correlation for correlation functions can be considered by the induction method.

In the case of s time correlation functions:

$$\langle \mathcal{H}_1(t_1) \mathcal{H}_2(t_2) \cdots \mathcal{H}_s(t_s) \rangle ,$$

where

$$\mathcal{H}_1(t_1) = \mathcal{A}_1^j(t_1) \cdots \mathcal{A}_p^j(t_j)$$

and $\mathcal{A}_p^j(t_j)$ is equal to $a_f(t)$ or to $a_f^\dagger(t)$; we find

$$\begin{aligned} &|\langle \mathcal{H}_1(t_1) \cdots \mathcal{H}_s(t_s) \rangle_\Gamma - \langle \mathcal{H}_1(t_1) \cdots \mathcal{H}_s(t_s) \rangle_{\Gamma_0}| \\ &\quad \leq \{Q_s(t_s - t_{s-1}) + \cdots + Q_s(t_2 - t_1) + Q\} \cdot \bar{\mathcal{E}}_2 \left(\frac{1}{V}, \delta \right) . \end{aligned} \tag{52}$$

Here $Q, Q_2, \ldots, Q_s$ are constants if $V \to \infty$.

We notice that it is possible to get in "x-representation" the analogous asymptotic inequalities for a large system ($V \to \infty$). Now we consider the correlation averages composed of the product of the field functions:

$$\begin{aligned} \Psi_\sigma(r,t) &= \frac{1}{\sqrt{V}} \sum_{(p)} a_{p\sigma}(t) e^{i(p\cdot r)}\,, \\ \Psi_\sigma^\dagger(r,t) &= \frac{1}{\sqrt{V}} \sum_{(p)} a_{p\sigma}^\dagger(t) e^{-i(p\cdot r)}\,. \end{aligned} \tag{53}$$

Here the summation runs as before over the set of quasi-discrete p. We consider averages composed of a product of operator field functions, constructed on the basis of model and trial Hamiltonians Γ and Γ_a, $\langle \psi_{\sigma_1}(r_1,t_1)\cdots\psi_{\sigma_s}(r_s,t_s)\rangle_{\Gamma,\Gamma_a}$, where s is an even number (for odd s the averages are identically zero) and where

$$\varphi_\sigma(r,t) = \Psi_\sigma(r,t) \qquad \text{or} \qquad \Psi_\sigma^\dagger(r,t).$$

By virtue of Eq. (53) we can write

$$\varphi_\sigma(r,t) = \frac{1}{\sqrt{V}} \sum_{(p)} \mathcal{A}_{p\sigma}(t) e^{i\mathcal{E}(p\cdot r)}\,,$$

where

$$\mathcal{A}_{p\sigma}(t) = a_{p\sigma}(t), \qquad \mathcal{E} = -1 \qquad \text{if} \qquad \varphi = \Psi^\dagger\,,$$

thus

$$\begin{aligned} &\langle \varphi_{\sigma_1}(r_1,t_1)\cdots\varphi_{\sigma_s}(r_s,t_s)\rangle_{\Gamma,\Gamma_a} \\ &\quad = \frac{1}{V^{\frac{s}{2}}} \sum_{p_1\ldots p_s} \langle \mathcal{A}_{p_1\sigma_1}(t)\cdots\mathcal{A}_{p_s\sigma_s}(t)\rangle_{\Gamma,\Gamma_a} e^{i\{\mathcal{E}_1(p_1\cdot r_1)+\cdots+\mathcal{E}_s(p_s\cdot r_s)\}}\,. \end{aligned} \tag{53a}$$

Let us consider the difference:

$$\langle \varphi_{\sigma_1}(r_1,t_1)\cdots\varphi_{\sigma_s}(r_s,t_s)\rangle_{\Gamma} - \langle \varphi_{\sigma_1}(r_1,t_1)\cdots\varphi_{\sigma_s}(r_s,t_s)\rangle_{\Gamma_a}$$

and show that it tends to zero or ($V \to \infty$). Recall that in this theory of generalized functions the following formula

$$\begin{aligned} F_V(r_1,\ldots,r_s) &\to F(r_1,\ldots,r_s)\,, \\ F(r_1,\ldots,r_s) &= \lim_{V\to\infty} F_V(r_1,\ldots,r_s)\,, \end{aligned} \tag{54}$$

is called generalized convergence. Consider the class $C(q,\nu)$ (where q and ν are positive numbers) of continuous and continuously differentiable functions $h(r_1,\ldots,r_s)$ such that

$$\{|r_1| + \cdots + |r_s|\}^j_{j=0,1,2,\ldots,\nu}\, |h(r_1,\ldots,r_s)| \le \text{const.}$$

and

$$\{|r_1| + \cdots + |r_s|\}^j \left| \frac{\partial^{q_1+\cdots+q_{3s}} h}{\partial X_{1,\alpha}^{q_1}\cdots\partial X_{3s,\alpha_l}^{q\,_{3s}}} \right| \le \text{const.}$$

$$(j = 0, 1, 2, \ldots, \nu; q_1 + \cdots + q_{3s} = 0, 1, \ldots, q; \alpha_l = 1, 2, 3)$$

in the space E^s of points $(r_1, \ldots, r_s)$. Here

$$(X_{l,1}, X_{l,2}, X_{l,3}) = r_l\,.$$

Then, if we fix positive numbers q and ν in such a way that the relation

$$\int h(\bar{r}_1, \ldots, \bar{r}_s) F_V(\bar{r}_1, \ldots, \bar{r}_s) d\bar{r}_1 \cdots d\bar{r}_s \to \int h(\bar{r}_1, \ldots, \bar{r}_s) F(\bar{r}_1, \ldots, \bar{r}_s) d\bar{r}_1 \cdots d\bar{r}_s \qquad (V \to \infty)$$

is valid for any functions $h(\bar{r}_1, \ldots, \bar{r}_s)$ from the class $C(q, \nu)$ there is said to be generalized convergence (54). For our purposes we shall confine ourselves to examining the class $\mathcal{L} = \mathcal{C}(\mathrm{II}, \nu)$ in which the numbers q_1 and ν_1 are chosen in such a way that the Fourier transform of the functions $h(\bar{r}_1, \ldots, \bar{r}_s)$:

$$\tilde{h}(p_1, \ldots, p_s) = \int h(\bar{r}_1, \ldots, \bar{r}_s) e^{i(\bar{p}_1 \cdot \bar{r}_1 + \cdots + \bar{p}_s \cdot \bar{r}_s)} d\bar{r}_1 \cdots d\bar{r}_s$$

is a continuous function of $p_1, \ldots, p_s$ in the whole space E^s, and satisfies the inequality:

$$\begin{aligned} \left|\tilde{h}(p_1, \ldots, p_s)\right| &\le \frac{\mathcal{K}_h}{\prod_{j=1}^{s} (|p_j|^2 + M_h)}\,, \\ \mathcal{K}_h &= \text{const.} > 0\,, \\ M_h &= \text{const.} > 0\,. \end{aligned} \tag{55}$$

It is possible to show for any function $h(\bar{r}_1, \ldots, \bar{r}_s)$ in the class $\mathcal{L}$:

$$\begin{aligned} &\int h(\bar{r}_1, \ldots, \bar{r}_s) \langle \varphi_{\sigma_1}(r_1, t_1) \cdots \varphi_{\sigma_s}(r_s, t_s) \rangle_\Gamma d\bar{r}_1 \cdots d\bar{r}_s \\ &\quad - \int h(\bar{r}_1, \ldots, \bar{r}_s) \langle \varphi_{\sigma_1}(r_1, t_1) \cdots \varphi_{\sigma_s}(r_s, t_s) \rangle_{\Gamma_0} d\bar{r}_1 \cdots d\bar{r}_s \to 0 \\ &\qquad \text{as } V \to \infty\,. \end{aligned}$$

Re-expressing the above-mentioned expression with the use of (53a) it is possible to prove that

$$\begin{aligned} &\frac{1}{V^{\frac{s}{2}}} \sum_{p_1, \ldots, p_s} \tilde{h}(\mathcal{E}_1 p_1, \ldots, \mathcal{E}_s p_s) \{ \langle \mathcal{A}_{p_1 \sigma_1}(t_1) \cdots \mathcal{A}_{p_s \sigma_s}(t_s) \rangle_\Gamma \\ &\qquad - \langle \mathcal{A}_{p_1 \sigma_1}(t_1) \cdots \mathcal{A}_{p_s \sigma_s}(t_s) \rangle_{\Gamma_a} \} \to 0\,. \end{aligned} \tag{56}$$

Here the summation runs over the points $p_1, \ldots, p_s$ of the quasi-discrete set E_V^s. We shall thereby also prove that in the generalized sense:

$$\langle \varphi_{\sigma_1}(r_1, t_1) \cdots \varphi_{\sigma_s}(r_s, t_s) \rangle_\Gamma - \langle \varphi_{\sigma_1}(r_1, t_1) \cdots \varphi_{\sigma_s}(r_s, t_s) \rangle_{\Gamma_a} \to 0 \quad \text{as } V \to \infty\,. \tag{57}$$

The question may arise why considering the averages of products of an operator field function we have proven only the generalized convergence (57). The point is that, even at fixed volume V, the sums representing the actual averages can, generally speaking, be divergent in the usual sense. Thus even in the simple case we can see that the average:

$$\langle \Psi_\sigma(r_1,t_1)\Psi_\sigma^\dagger(r_2,t_2)\rangle_{\Gamma_a} = \frac{1}{V}\sum_{p_1,p_2} \langle a_{p_1\sigma_1}(t_1)\cdot a^\dagger_{p_2\sigma_2}(t_1)\rangle_{\Gamma_a} e^{i\{(p_1\cdot r_1)-(p_2\cdot r_2)\}}$$

is convergent only in the generalized sense, and this expression is defined only as a generalized function of r_1, r_2 even for finite volume V.

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