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Interlayer effects in the newest high-T_c superconductors

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The theory of the dependence of T_c on the number of parallel square planar CuO_2 layers in unit cells of the newest bismuth and thallium superconductors is developed. It is shown that the observed nonmonotonic dependence with the maximum at some layers can be explained only if one takes into account the chaining of intraplanar distances, arising with the generation of bisolitons.

1. Introduction

Almost one year after the discovery of yttrium oxide superconductors three new series of superconductors, bismuth-based and thallium-based, were obtained. In 1988 Maeda et al. [1] discovered the drop of the resistance in the crystal $Bi_2Sr_2Cu_2O_{7+y}$ at 120 K. This result was repeated by Chu with collaborators [2]. During a short time after that Sheng and Herman [3] have synthesized three thallium oxides: $Tl_2Ba_2Cu_1O_8$, $Tl_2Ca_1Ba_2Cu_2O_8$ and $Tl_2Ca_2Ba_2Cu_3O_{10}$ containing one, two and three CuO_2 planes, separated by Tl–O bilayers. In the same year Parkin et al. [4] discovered three more thallium oxides, containing in a unit cell the CuO_2 plane layers separated by monolayers. Tl–O: $Tl_1Ba_2Cu_1O_5$, $Tl_1Ca_1Ba_2O_7$ and $Tl_1Ca_2Ba_2Cu_3O_9$. Later the new members of these three families were discovered.

In these newly discovered bismuth and thallium high-T_c superconductors there exist some ($N=1, 2, 3, \cdots$) square-planar CuO_2 layers, which are parallel to the a–b axes of the crystal. In bismuth and thallium oxides, $Bi_2Sr_2Ca_{N-1}Cu_NO_{2N+4}$ and $Tl_2Ba_2Ca_{N-1}Cu_NO_{2N+4}$ they are sandwiched between double layers of bismuth or thallium oxides. In the family of thallium oxides $Tl_1Ba_2Ca_{N-1}Cu_NO_{2N+3}$ of square-planar CuO_2 layers they are sandwiched between single layers of thallium oxide. The neighbouring CuO_2 layers in a sandwich are separated by Ca(Sr) ions.

For a shorthand notation of the bismuth and thallium superconductors with N CuO_2 layers one needs the symbols [5]: $Bi(2:2:(N-1):N)$ and $Tl(m:2:(N-1):N)$ ($m=1, 2$) where the four digits in the brackets represent the number of cations in the unit cell in the order Bi(Tl), Ca, (Sr, Ba), Cu. For example the abbreviation for $Tl_1Ca_2Ba_2Cu_3O_9$ will be Tl(1:2:2:3).

High-T_c superconductivity appears when doping charge carries (holes) into the CuO_2 sheets. The other cations such as Tl, Ca, Bi, Sr are playing the roles of "spacers" which stipulate the space structure of the unit cell. They also are the donors of charge carriers in the plane CuO_2 sheets.

Figure 1 represents a schematic distribution of the ions in the unit cell of the superconductors $Tl_1Ba_2Ca_{N-1}Cu_NO_{2N+4}$ or for short $1:2:N-1:N$.

The conductivity, i.e. the density of the carriers, in these newly discovered superconductors is much higher in the plane of the layers than in perpendicular direction. In general, the carrier densities are low (~a few times 10^{21} cm^{-3}) and their motion is bounded in layers of conductivity in the unit cell.

There is a very weak connection between the layers. These materials are extremely type II superconductors with very short anisotropic coherence lengths: $\xi_{a,b} \sim 10$–15 Å, $\xi_c \sim 3$–5 Å and large critical magnetic fields (H_{cr}).

Within each of these families the superconducting transition temperature T_c increases with the number N of layers in the sandwich. So, many authors express the opinion that the essential part of the superconductivity of these oxides is played by the sandwiches in the square-planar CuO_2.

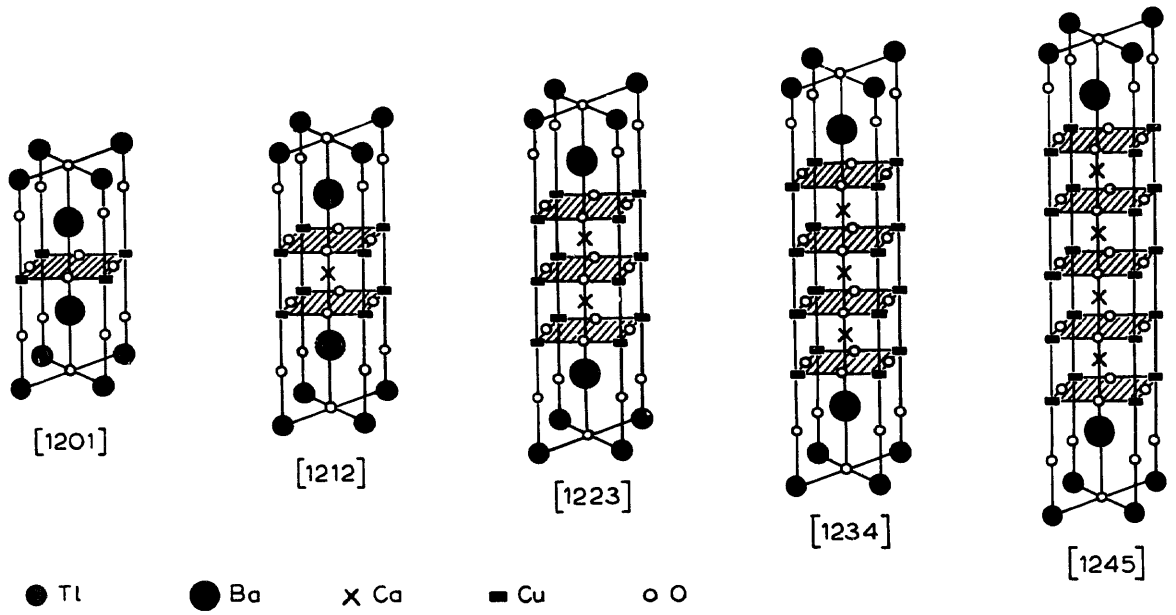

Fig. 1. The structure of the unit cells of the family of thallium oxides $Tl_1Ba_2Ca_{N-1}Cu_NO_{2N+3}$ at values of N equal to 1, 2, 3, 4 and 5.

Having discovered that T_c is growing with increase of the number of CuO_2 layers induces the hope to obtain a high T_c on the basis of the synthesis of the samples having a larger number of layers. If this increase is linear, at about N equal to 10 the critical temperature will be comparable with room temperature. But in reality this is not the case. For example, in the bismuth series a change of N from one to two makes the temperature T_c increase with 60 K, a transfer from two to three leads only to 30 K, a transfer from three to four yields an increase of 14 K.

In the phenomenological theoretical investigation of the dependence of T_c on the number of $Cu-O_2$ layers per unit cell in the new families of high-T_c superconductors one usually utilizes the following simplification:

(1) one assumes that binding between the sandwiches of CuO_2 layers of neighbouring unit cells separated by thallium and bismuth oxides is very weak;

(2) only the CuO_2 planes in every sandwich are active in determining T_c;

(3) take only into account the coupling between nearest neighbour CuO_2 layers in a sandwich are separated by Ca or Sr ions;

(4) all the layers in sandwiches are taken to be identical.

The first theoretical investigation of the T_c's dependence on the number of CuO_2 layers inside the unit cell has been done by Anderson et al. [5] on the basis of a resonating valence bond model.

Birman and Lu [6] employed the approximation of Ginsburg–Landau to the newest high-T_c superconductors. They find the upper T_c limit of 140 K for monolayer and bilayer thallium series.

In some papers the dependence of T_c on the number of layers CuO_2 per unit cell was investigated by using a microscopic formulation of the generalized BCS pairing. In the paper written by Eab and Tang [7,8] on the basis of the phenomenological model of Ginsburg–Landau by means of minimization of the energy an expression for $T_c(N)$ was obtained. Using the experimental value of T_c for the first members of the series one finds T_c for higher members.

A theory for a layered crystal with an arbitrary number of interacting layers inside the unit cell has been developed by Jha [9] (see also refs. [10,11]). In all of these papers it seemingly was demonstrated that T_c monotonically increases with the number of layers in the sandwiches and passes to a constant limit when N is larger than ten. But the experiments carried out in 1989 by Kikuchi, Nakajima et al. [12] have shown that this is not actually the case. They have synthesized the high-T_c superconductor $Tl_2Ba_2Ca_2Cu_4O_{12}$ and discovered the same lowering

Table 1
Single Tl layered system $Tl_1Ba_2Ca_{N-1}Cu_NO_{2N+3}$ [12–18]

N	1	2	3	4	5	6
a (Å)	-	3.8500	3.8493	3.8153	3.8469	-
T_c (K)	13–15	78–91	116–120	122	106	102

Table 2
Double Tl layered system $Tl_2Ba_2Ca_{N-1}Cu_NO_{2N+4}$ [19–21]

N	1	2	3	4
a (Å)	3.8587	3.857	3.822	-
T_c (K)	20–80	110	125	108–112

Table 3
Double Bi layered system $Bi(Ca, Cr)_{N+1}Cu_NO_{2N+4}$ [22–24]

N	1	2	3	4
a (Å)	3.796	3.823	3.818	-
T_c (K)	12–22	85–90	110–120	90

of the temperature T_c under the passage from three to four layered specimens. They also have synthesized high-T_c superconductors $Tl_1Ba_2Ca_{N-1}Cu_NO_{2N+3}$ with N equal from two to five [13] and have shown that T_c increases with the number of layers N equal to four and then is lowering at N equal to five and six.

The T_c (K) dependence on the intraplanar lattice constant a (Å) on the number of layers in unit cell is given in tables 1, 2 and 3. The highest T_c value was obtained for $N=4$ which has four CuO_2 layers in a unit cell.

In both single and double Tl layered systems, the systems with the highest T_c ($N=4$ for the former and $N=3$ for the latter) have the shortest intraplanar lattice constant a (Å). In this case too the system with the highest T_c ($N=3$) has the shortest a (Å).

In the following sections we will give the explanation of the T_c lowering for larger N as found experimentally on the basis of a nonlinear bisoliton model which was developed at the Institute for Theoretical Physics in Kiev [25–30].

2. Condensate bisolitons in superconductors having a single CuO_2 layer in the unit cell

According to a bisoliton model high-T_c superconductivity [25–29] in lanthanum and yttrium superconductors containing in their unit cells a single CuO_2 layer situated parallel to the a, b planes of the crystal the superconductivity is stipulated by quasi-one-dimensional condensate bisolitons [21]. The bisolitons are distributed periodically in these layers along the axes a or b. One bisoliton contained in each period aL is formed by two quasi-particles (usually holes). Quasi-particles are connected in a singlet spin state by a local deformation field.

In a condensate moving with the velocity $V=\hbar k/m$, much smaller than that of longitudinal sound, V_0, in a formation of bisoliton two quasi-particles with effective masses m and wave numbers $k_1=2k+k_F$ and $k_2=-k_F$ participate.

In the frame $\xi=(x-Vt)/a$, moving with velocity V, in each period aL, containing one bisoliton symmetric wave, the function of the condensate of bisolitons has the form

$$\Psi(\xi_1, \xi_2, t)=\sqrt{2}\,\phi(\xi_1)\phi(\xi_2)\cos[k_F(\xi_1-\xi_2)]e^{ik(\xi_1+\xi_2)-i\delta_p t}. \quad (2.1)$$

The real periodic functions, $\phi(\xi)=\phi(\xi+L)$ satisfy the normalization condition

$$\int_0^L \phi^2(\xi)\mathrm{d}\xi=1.$$

These are the solutions to a nonlinear Schrödinger equation,

$$\left[\frac{\partial^2}{\partial\xi^2}+4g\phi^2(\xi)-\epsilon\right]\phi(\xi)=0, \quad \epsilon=\delta_p/J, \quad (2.2)$$

determining the motion of quasi-particles in the field of deformation

$$U(\xi) = -4g\phi^2(\xi) . \tag{2.3}$$

Here $\mathcal{E}_p$ is the energy of the quasi-particle, $g=\sigma^2/2\mathcal{K}J$ is a nondimensional parameter of the interaction between a quasi-particle and the field of deformation; σ is the electron–phonon interaction; $\mathcal{K}$ the coefficient of elasticity at a chain.

$J=\hbar^2/2ma^2$ is the energy of the exchange interaction at neighbouring unit cells; m is effective mass of a quasi-particle appearing when doping the crystal; a is a lattice constant.

In the general case the periodic solution eq. (2.2) is expressed by means of the elliptic Jacobi functions. Provided that the inequality is fulfilled $gL \gg 1$, these functions are reduced to hyperbolic functions

$$\phi(\xi) = \sqrt{g/2}\ \mathrm{sech}\ (g\xi) . \tag{2.4}$$

In this case the energy of both quasi-particles in the field of deformation (2.3), counted from the Fermi energy E_F, has the value

$$\mathcal{E}_p = -g^2 J . \tag{2.5}$$

Under the creation of bisolitons there takes place the local displacements of lattice sites na with the value β_n so the change of distances between neighbouring sites is defined by $\rho_n = \beta_n - \beta_{n+1}$. In the continuum approximation one can write

$$[\beta_n \to \beta(\xi)]$$

$$[\beta_{n\pm1} \to \beta(\xi) \pm a\partial\beta/\partial\xi + \tfrac{1}{2}a^2\partial^2\beta/\partial\xi^2] .$$

Therefore

$$\rho_n \to \rho(\xi) = -a\partial\beta/\partial\xi .$$

The energy of the longitudinal local deformation of the lattice in one period has the value

$$W_\parallel = \tfrac{1}{2}\mathcal{K}\int_0^L \rho^2(\xi)\,\mathrm{d}\xi = \tfrac{1}{3}g^2 J . \tag{2.6}$$

The total energy of a bisoliton at rest, including the energy of the longitudinal deformation, eq. (2.6), is defined by

$$E_{bs} = \mathcal{E}_p + W_\parallel = -\tfrac{2}{3}g^2 J . \tag{2.7}$$

With the creation of a bisoliton a local deformation takes place. We define an average diminuation $\ll\rho\gg$ of a lattice constant via the equality

$$\Delta a \equiv \ll\rho\gg = (2W_\parallel/\mathcal{K})^{1/2} .$$

Thus, taking into account eqs. (2.6) and (2.7) the value $\ll\rho\gg$ may be expressed by means of the total energy of a bisoliton, eq. (2.7), by

$$\ll\rho\gg = \left(\frac{1}{\mathcal{K}}|E_{bs}|\right)^{1/2} . \tag{2.8}$$

3. Interlayer effect in high-T_c superconductors

In the newly discovered thallium and bismuth high-T_c superconductors there are some (two, three, etc.) square-planar CuO_2 layers. These are sandwiched between layers of thallium or bismuth oxides.

In the theoretical investigation of a bisoliton condensate of these superconductors one usually utilizes the assumption that only the CuO_2 layers determine the T_c value.

Let us assume that there are N square-planar CuO_2 layers in the unit cell. The energy pairs of quasi-particles make up the immobile condensate bisolitons when there is interaction with longitudinal $\beta_{n\alpha}$ (along the layers) and transversal $\xi_{n\alpha}$ displacements of the sites in the crystal, characterized by the Hamiltonian

$$\begin{aligned} H = \sum_{n\alpha} \varphi_{n\alpha}\{ & -J[\varphi_{n+1,\alpha} + \varphi_{n-1,\alpha}] \\ & + [W_\perp + W_\parallel + 2\sigma_\parallel(\beta_{n+1} - \beta_n)]\varphi_{n\alpha} \\ & - \mathcal{L}(\varphi_{n,\alpha+1} + \varphi_{n,\alpha-1}) \\ & + 2\sigma_\perp[(\xi_{n\alpha} - \xi_{n,\alpha+1})\varphi_{n,\alpha+1} \\ & + (\xi_{n\alpha} - \xi_{n,\alpha-1})\varphi_{n,\alpha-1}]\} . \end{aligned} \tag{3.1}$$

The subscript $\alpha = 1, 2, \ldots, N$ is indexing the layers in the unit cell. The subscript n changes from 1 to L. The function $\varphi_{n\alpha}$ satisfies the periodic condition

$$\varphi_{n\alpha} = \varphi_{n+1,\alpha} \tag{3.2a}$$

and boundary conditions

$$\varphi_{n0} = \varphi_{n,N+1} = 0 . \tag{3.2b}$$

The normalization conditions

$$\sum_{n=1}^{L}\sum_{\alpha=1}^{N} \varphi_{n\alpha}^2 = 1 \tag{3.3}$$

show that there is one bisoliton in a sandwich.

The energy of a longitudinal deformation is defined by the formula

$$W_{\parallel}=\tfrac{1}{2}\mathscr{H}_{\parallel}\sum_{n\alpha}(\beta_{n+1,\alpha}-\beta_{n\alpha})^2\,, \tag{3.4}$$

where $\mathscr{H}_{\parallel}$ is an elasticity constant. The energy of a transversal deformation is defined by

$$W_{\perp}=\tfrac{1}{2}\mathscr{H}_{\perp}\sum_{n\alpha}(\xi_{n\alpha}-\xi_{n,\alpha-1})^2\,. \tag{3.5}$$

Varying eq. (3.1) in turn on the longitudinal and transversal displacements we obtain

$$\rho_{n\alpha}=\beta_{n-1,\alpha}-\beta_{n\alpha}=-2\sigma|\varphi_{n\alpha}|^2/\mathscr{H}_{\parallel}$$
$$\xi_{n,\alpha+1}-\xi_{n,\alpha}=2\sigma_{\perp}\varphi_{n\alpha}\varphi_{n,\alpha+1}/\mathscr{H}_{\perp}\,. \tag{3.6}$$

By substituting eqs. (3.6) into eq. (3.1) we obtain

$$H=J\sum_{n\alpha}\varphi_{n\alpha}\Big\{\frac{W_{\parallel}+W_{\perp}}{J}\varphi_{n\alpha}-[\varphi_{n+1,\alpha}-\varphi_{n-1,\alpha}]$$
$$-4g\varphi_{n\alpha}^3-\gamma(\varphi_{n,\alpha+1}-\varphi_{n,\alpha-1})$$
$$-\delta[\varphi_{n,\alpha+1}^2+\varphi_{n,\alpha-1}^2]\varphi_{n\alpha}\Big\}\,, \tag{3.7}$$

where

$$\gamma=\mathscr{L}/J\,,\quad g_{\parallel}^2=\sigma_{\parallel}^2/2\kappa J\,,\quad 2\delta=\sigma_{\perp}^2/\mathscr{H}J\,. \tag{3.8}$$

Going over to the continuum approximation

$$an\to a\xi\,,\quad \varphi_{n\alpha}\to\varphi_{\alpha}(\xi)\,,$$
$$\varphi_{n+1,\alpha}\to\varphi_{\alpha}(\xi)\pm\partial\varphi/\partial\xi+\tfrac{1}{2}\partial^2\varphi/\partial\xi^2+\cdots\,,$$
$$\beta_{n\pm1,\alpha}=\beta_{\alpha}(\xi)\pm\partial\beta_{\alpha}(\xi)/\partial\xi+\tfrac{1}{2}\partial^2\beta_{\alpha}(\xi)/\partial\xi^2+\cdots\,.$$

With regard to eqs. (3.3), (3.4) and (3.5) we can transform eq. (3.1) to

$$\mathscr{E}\equiv\frac{H}{J}=\sum_{\alpha}\int_{-L/2}^{L/2}\mathrm{d}\xi\,\varphi_{\alpha}(\xi)$$
$$\times\Big\{-\frac{\partial^2}{\partial\xi^2}+4g\varphi_{\alpha}(\xi)\Big\}\varphi_{\alpha}(\xi)$$
$$+\frac{(W_{\perp}+W_{\parallel})}{J}\varphi_{\alpha}^2(\xi)-\gamma[\varphi_{\alpha+1}(\xi)+\varphi_{\alpha-1}(\xi)$$
$$-\delta[\varphi_{\alpha+1}^2(\xi)+\varphi_{\alpha-1}^2(\xi)]\Big\}\varphi_{\alpha}(\xi)\,. \tag{3.9}$$

The energy of the deformations eqs. (3.4) and (3.5) is defined by

$$W_{\parallel}=\frac{2\sigma_{\parallel}^2}{\mathscr{H}_{\parallel}J}\sum_{\alpha}\int_{-L/2}^{L/2}|\varphi_{\alpha}(\xi)|^4\,\mathrm{d}\xi\,,$$

$$W_{\perp}=\frac{2\sigma_{\perp}^2}{\mathscr{H}_{\perp}J}\sum_{\alpha}\int_{-L/2}^{L/2}|\varphi_{\alpha}(\xi)|^4\,\mathrm{d}\xi\,. \tag{3.10}$$

Let $\tilde{\varphi}_{\alpha}(\xi)$ be the wave function of a condensate bi-soliton in an isolated CuO_2 layer, which satisfies the nonlinear Schrödinger equation

$$\Big[\frac{\partial^2}{\partial\xi^2}+4g\tilde{\varphi}_{\alpha}^2(\xi)\Big]\tilde{\varphi}_{\alpha}(\xi)=-g_{\parallel}^2\tilde{\varphi}_{\alpha}(\xi)\,, \tag{3.11}$$

and which satisfies the normalization condition

$$\int_{-L/2}^{L/2}\tilde{\varphi}_{\alpha}^2(\xi)\mathrm{d}\xi=1\,, \tag{3.12}$$

we have

$$\tilde{\varphi}_{\alpha}(\xi)=\sqrt{g/2}\,\mathrm{sech}(g\xi)\,. \tag{3.13}$$

The wave function of a sandwich which contains N layers may be looked for in the form

$$\Psi(\xi)=\sum_{\alpha=1}^{N}C_{\alpha}\tilde{\varphi}_{\alpha}(\xi)\,, \tag{3.14}$$

with the coefficients C_{α} satisfying the conditions

$$C_0=C_{N+1}=0\,,\quad \sum_{\alpha=1}^{N}C_{\alpha}^2=1\,. \tag{3.15}$$

By substituting in eq. (3.9) the function

$$\varphi_{\alpha}(\xi)=C_{\alpha}\tilde{\varphi}_{\alpha}(\xi) \tag{3.16}$$

we obtain a system of equations which determines the coefficients C_{α}.

For simplification of the calculations we use the following approximate expressions:

$$W_{\parallel}=g_{\parallel}^2/3J\,,\quad W_{\perp}=g_{\perp}^2/3J\,.$$

In this case the equality (3.7) will be transformed to

$$\sum_{\alpha=1}^{N}C_{\alpha}[(\mathscr{E}_{\mathrm{bs}}^0-\mathscr{E}_{\mathrm{bs}}(N)+\delta(C_{\alpha+1}^2$$
$$+C_{\alpha-1}^2)C_{\alpha}+\gamma(C_{\alpha+1}+C_{\alpha-1})]=0\,, \tag{3.17}$$

where

$$\mathscr{E}_{\mathrm{bs}}^0=-\tfrac{2}{3}g_{\parallel}^2+\tfrac{1}{3}\delta^2\,, \tag{3.18}$$

From eq. (3.17) it follows that the energy $\mathscr{E}_{\mathrm{bs}}(N)$ of the N CuO_2 layers in a sandwich is determined by a system of equations

$$X(N)C_\alpha-\gamma[C_{\alpha+1}+C_{\alpha-1}]$$
$$-\delta[C^2_{\alpha+1}+C^2_{\alpha-1}]C_\alpha=0 \quad (3.19)$$

where

$$X(N)\equiv\mathcal{E}_{bs}(N)-\mathcal{E}^0_{bs}. \quad (3.20)$$

The second term in eq. (3.19) takes into account the exchange interaction between neighbour CuO_2 planes separated by ions Ca(Sr). The third term takes into account the role of the changing of interplane distances.

If in a unit cell there is only a single CuO_2 layer in eq. (3.18) we must put γ and δ equal to zero. In this case an energy of bisoliton condensate $\mathcal{E}_{bs}(1)$ coincides with expression (2.7).

4. Energy of sandwich CuO_2 layers under fixed distances between them

If we do not take into account the change in the distances between layers in sandwiches, the value of δ in the system of equations (3.20) must be zero. So it reduces to the equation

$$X(N)C_\alpha-\gamma[C_{\alpha+1}+C_{\alpha-1}]=0, \quad (4.1)$$

with additional conditions eq. (3.15). In this case because of the interaction between the layers the energetic level $\mathcal{E}^0_{bs}$ of a single layer is split in N sublevels. The energies of these sublevels are defined by the determinant which contains N rows and columns

$$\begin{pmatrix} X(N) & -\gamma & 0 & 0 & & & \\ -\gamma & X(N) & -\gamma & 0 & & & \\ & & & & -\gamma & X(N) & -\gamma \\ & & & & & -\gamma & X(N) \end{pmatrix}=0. \quad (4.2)$$

The N roots of this equation have values

$$X_j(N)=-2\gamma\cos[\pi j/(N+1)], \quad j=1,\ldots,N. \quad (4.3)$$

In this case the coefficients C_α in the sum (3.14) have the value

$$C^{(j)}_\alpha(N)=\sqrt{\frac{2}{N+1}}\sin[\pi\alpha j/(N+1)], \quad j=1,\ldots,N. \quad (4.4)$$

Therefore, the bisoliton state j in a single period L is distributed among the layers with the probability

$$w_j=[C^j_\alpha(N)]^2=\frac{2}{N+1}\sin^2\frac{\pi\alpha j}{N+1}, \quad \sum_{\alpha=1}^{N}C^2_\alpha(N)=1. \quad (4.5)$$

The wave function (3.14) of this state is defined by the formula

$$\Psi_j(N)=\sum_{\alpha=1}^{N}\left(\frac{2}{N+1}\right)^{1/2}\tilde{\varphi}_\alpha(\xi)\sin[\pi\alpha j/(N+1)]. \quad (4.6)$$

It corresponds to the energy

$$\mathcal{E}^{(j)}_{bs}(N)=\mathcal{E}^{(0)}_{cs}-2\gamma\cos[\pi j/(N+1)], \quad (4.7)$$

where $\mathcal{E}^{(0)}_{bs}$ is the energy of a single layer. The superconductive state is defined by the lower value of j equal to one

$$\mathcal{E}^{(1)}_{bs}(N)=\mathcal{E}^{(0)}_{bs}-2\gamma\cos[\pi/(N+1)]. \quad (4.8)$$

So the critical temperature of the N-layered superconductor is defined by the formula

$$T_c(N)-T_c(1)=A2\gamma\cos[\pi/(N+1)], \quad (4.9)$$

where $T_c(1)$ is the critical temperature of a single layer phase, and the coefficient A can be defined from experimental data of the first members of the series.

5. Energy of sandwiches of CuO_2 layers with regard to the changing of the distances between the layers

For an evaluation of the energy of the sandwiches of bisolitons in the first approximation we shall perform in eq. (3.18) the substitution

$$C^2_{\alpha+1}+C^2_{\alpha-1}\to D(N). \quad (5.1)$$

In this case the system of equations (3.20) is transformed to

$$[X(N)-\delta D(N)]C_\alpha-\gamma[C_{\alpha+1}+C_{\alpha-1}]=0. \quad (5.2)$$

As one more further simplification the value $D(N)$ is estimated by using C_α obtained in section 4 for the value $\delta=0$.

Table 4

N	2	3	4	5	6	7	10	12	15
$2\cos\pi/(N+1)$	1.0	1.41	1.62	1.73	1.80	1.85	1.90	1.94	1.96
$D(N)$	1.0	0.66	0.55	0.30	0.17	0.10	0.08	0.02	0.01

Table 5
Dependence of $\tau(N)$ on numbers N of sublevel in the sandwich

δ/γ	N					
	2	3	4	5	6	7
0.6	1.60	1.80	1.95	1.91	1.90	1.91
0.8	1.80	2.15	2.06	1.97	1.93	1.93

Under the substitution (5.1) the solution to eq. (3.20) has the form, for $j=1$,

$$X(N)=-\gamma[2\cos\pi/(N+1)+\frac{\delta}{\gamma}D(N)]\,. \qquad (5.3)$$

The critical temperature of N-layered superconductors will be defined by the expression

$$\tau(N)\equiv\frac{T_c(N)-T_c(1)}{\gamma A}$$

$$=2\cos\pi/(N+1)+\frac{\delta}{\gamma}D(N)\,. \qquad (5.4)$$

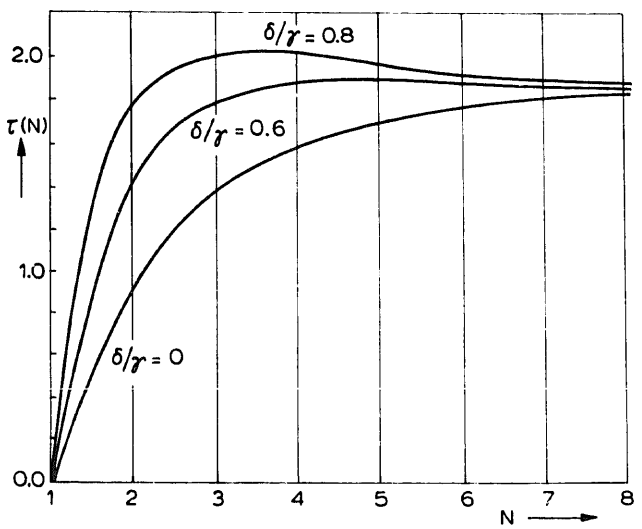

Fig. 2. The dependence of the ratio $\tau(N)=(T_c(N)-T_c(1))/A\sigma$ at values σ/γ equal to 0, 0.6, 0.8 vs. the number (N) of parallel CuO_2 layers in the unit cells of layered superconductors.

The dependence of the function $2\cos\pi/(N+1)$ and $D(N)$ on N is shown in table 4.

The function $D(N)$ is described by the average distribution of a probability of the presence of one bisoliton on a single layer of the N layers sandwich. Naturally the value $D(N)$ is diminished when the number of N sublevels in the sandwich is increased.

Using the values of table 4 we obtain the function $\tau(N)$ which is proportional to T_c. The values of $\tau(N)$ for two values of δ/γ, 0.6 and 0.8, are shown in table 5 as a function of the number of layers (N).

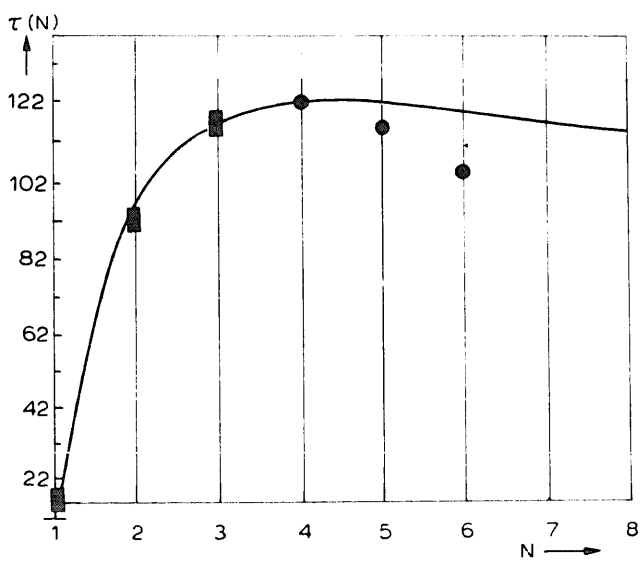

Fig. 3. The theoretical dependence of T_c (having $\sigma/\gamma=0.6$) for the family of thallium oxides $Tl_1Ba_2Ca_{N-1}Cu_NO_{2N+3}$ from the number N of parallel CuO_2 layers in the unit cells. The circles show the experimental data cited in table 1.

According to eq. (2.8) the maximal value $\tau(N)$ corresponds to maximal critical temperature and minimal average value intraplanar lattice constant a (Å). These theoretical results correspond to the experimental data shown in tables 1, 2 and 3.

In fig. 2 the dependence of $\tau(N)$ on N is shown for the values δ/γ equal to 0, 0.6 and 0.8. The presence of the maximum value on the curve $\tau(N)$ is an obvious consequence of taking into account the displacement of the intraplanar distance in the sandwich.

Using the experimental values $T_c(N)$ for the family of superconductors $Tl_1Ba_2Ca_{N-1}Cu_NO_{2N+3}$ one can obtain at $\delta/\gamma=0.6$ the theoretical dependence of T_c on the number of CuO_2 layers in the unit cell. This dependence is shown by the curve in fig. 3. The experimental values are indicated by circles.

References

[1] H. Maeda, Y. Tanaka, M. Fukutomi and T. Asano, Jpn. J. Appl. Phys. 27 (1988) L209.
[2] C.W. Chu, J. Bechtold, L. Gao, P.H. Hoor and Z.J. Hung, Phys. Rev. Lett. 60 (1988) 941.
[3] Z.Z. Sheng, A.M. Herman, A. El Ali et al., Nature 332 (1988) 55, 138; ibid., Phys. Rev. Lett. 60 (1988) 937; ibid., Appl. Phys. Lett. 52 (1988) 1738.
[4] S.S. Parkin, V.Y. Loe, A.I. Nazzed et al., Phys. Rev. 38 (1988) 6531.
[5] J.M. Whently, T.C. Hsu and P.W. Anderson, Nature 333 (1988) 121; ibid., Phys. Rev. B 37 (1988) 5897.
[6] J. Birman and J.P. Lu, Phys. Rev. B 39 (1988) 2238.
[7] C.H. Eab and T.M. Tang, Phys. Rev. B 40 (1989) 4427; ibid., Phys. Lett. A 134 (1989) 253.
[8] C.H. Eab and I.M. Tang, Phys. Lett. A 134 (1989) 253.
[9] S.S. Jha, Phase Transition 19 (1989) 3.
[10] A. Nakamura, Jpn. J. Appl. Phys. 28 (1989) 1468.
[11] A.M. Sleight, 142 (1988) 1519.
[12] M. Kikuchi, S. Nakajima, Y. Syono, K. Hiraga, T. Oka, D. Shindo, N. Kobayashi, H. Iwasaki and Y. Muto, Physica C 158 (1989) 79.
[13] S. Nakajima, M. Kikuchi, Y. Syono, T. Oku, D. Shindo, K. Hiraga, N. Kobayashi, H. Iwasaki and Y. Muto, Physica C 158 (1989) 471.
[14] K. Hiraga, N. Kabayashi, H. Twasaki and Y. Muto, Jpn. J. Appl. Phys. 27 (1988) L5050.
[15] H. Ihara, R. Sagice, N. Hirabayashi, M. Terada, M. Jo, K. Hayashi, Negasashi, M. Jokomoto, Y. Kimura and T. Shimomura, Letters to Nature 333 (1989) 510.
[16] R. Pool, Science 240 (1988) 146.
[17] R.S. Beyers, S.S. Parkin, V.Y. Lee, et al., IBM J. Res. Dev. 23 (1989) 228.
[18] T. Kurakawa, X. Ampo and C. Tajama, Superconductivity: Physics, Chemistry, Technology 3 (1990) 773 (in Russian).
[19] H. Ihara, R. Sugise, Hiralayashi et al., Nature 334 (1988) 510.
[20] C.C. Torardi, M.A. Subramanian, J.C. Calabress et al., Science 240 (1988) 631; ibid., Nature 332 (1988) 420.
[21] M. Kikuchi, S. Nakajima, Y. Syono, K. Hiraga, D. Shindo, T. Oku and N. Kobayashi, Physica C 170 (1990) 443.
[22] J.L. Tallon, P.W. Bunkly, P.W. Gilbert et al., Nature 133 (1988) 153.
[23] B. Batlog, R.J. Cava, Schneemeyer and G.P. Espinosa, IBM J. Res. Dev. 35 (1989) 228.
[24] N.E. Flower, M.R. Presland and P.W. Gilbert, Physica C 165 (1990) 161.
[25] A.S. Davydov, Phys. Status Solidi B 146 (1988).
[26] A.S. Davydov and V.N. Ermakov, Phys. Status Solidi B 148 (1988) 305.
[27] A.S. Davydov, Nonlinearity 2 (1989) 383.
[28] A.S. Davydov, Supercond. Sci. Technol. 2 (1989) 153.
[29] A.S. Davydov, Phys. Rep. 190 (1990) 193.
[30] A.S. Davydov, Phys. Status Solidi B 160 (1990).