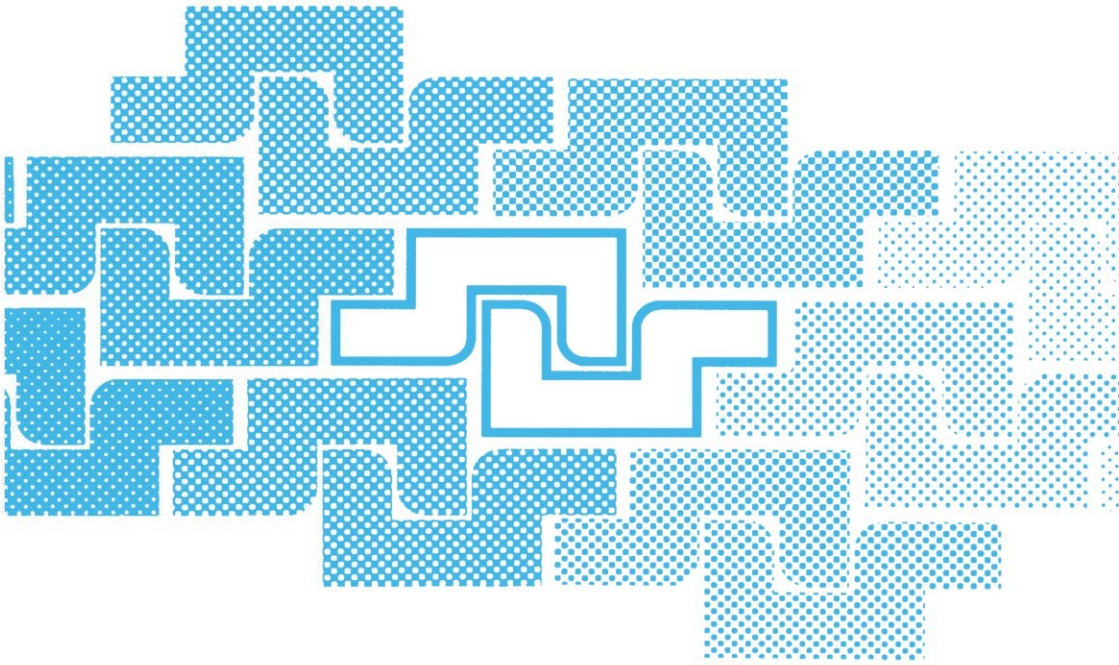

New Trends in Superconductivity

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STUDY OF DECOHERENCE TIME OF ELECTRONIC STATES IN QUANTUM DOTS, JOSEPHSON JUNCTIONS AND FRACTIONAL QUANTUM HALL EFFECT "PSEUDO-SPIN" QUANTUM COMPUTING DEVICES

MASANORI SUGAHARA
Graduate School of Engineering, Yokohama National University, Hodogaya, Yokohama, 240-8501, Japan

SERGEI KRUCHININ
Bogolyubov Institute for Theoretical Physics, Metrologichna str. 14-b, Kiev, 252143, Ukraine

NIKOLAI N. BOGOLUBOV, JR.
V.A. Steklov Mathematical Institute, Academy of Science of Russia, Gubkin str. 8, GSP-1, 117966, Moscow, Russia

1. Introduction

Recently quantum computing has attracted wide attention [1]. It was shown that any quantum computer can be composed based on the combination of "controlled NOT gates" [2,3], where the quantum state is processed using a control bit and target bit. In order that a gate acquires practical feasibility, the decoherence time $\tau_{\mathrm{d}} \equiv 1/\gamma$ of the quantum state has to be long enough for quantum-computing operations. For example, it has been estimated that the "resolution of the number N into factors" needs large numbers of computing processes of the order of $\tilde{}\exp[\gamma(\log N)^2]$ [4], or to take substantial computation time as long as $\tilde{}$40min for 2048 bits of N. [5] These estimations reveal that a quantum computer with enough reliability and superiority in comparison with traditional computer is feasible either at (i) "effectively zero temperature" in the hitherto-proposed gate systems or in a new system having the practical immunity from phonon disturbance. The authors (M.S and S.K) proposed [6] that we can make a controlled NOT gate with very long τ_{d} by the use of the pseudo-spin states in a two layer FQHE system.

In this paper we first make a comparison of τ_{d} between the cases of a the quantum dot of metal (or semiconductor), a superconductor junction, and in the fractional quantum Hall effect (FQHE) system based on the theoretical study of the decoherence by thermal phonon disturbance. After that we propose, based on the "pseudo-spin" state of the FQHE system, a controlled NOT gate with long τ_{d} and with adaptability to IC technology.

2. Calculation of Decoherence Time

2.1. QUANTUM DOT OF METAL (OR SEMICONDUCTOR)

In a metal (or a semiconductor) quantum dot we consider one electron which makes electron-phonon interaction through the deformation potential of the crystal lattice. When lattice vibration makes a deformation of a small lattice volume v to $v + \Delta v$, there appears electro-static potential $C_{\mathrm{d}}\Delta v/v \equiv C_{\mathrm{d}}\ div\ \mathbf{u}$, where $\mathbf{u}$ is the displacement of lattice coordinate from equilibrium position, and C_{d} is "deformation potential". In the free electron model of metals

$$C_{\mathrm{d}} = -\,(2/3)\,E_{\mathrm{F}}[\mathrm{J}],$$

where E_{F} is Fermi level. Based on the deformation-induced potential, the electron-phonon interaction Hamiltonian H' is given

$$\begin{aligned} H' &= \int |\psi_{\mathrm{e}}(\mathbf{r})|^2\, C_{\mathrm{d}} div \mathbf{u} d\mathbf{r} \qquad (1) \\ &= iC_{\mathrm{d}} \int |\psi_{\mathrm{e}}(\mathbf{r})|^2 \left(\frac{\hbar}{2Mn_{\mathrm{i}}V}\right)^{1/2} \sum_{\mathbf{k}} \frac{(\mathbf{e_k k})}{\sqrt{\omega_{\mathbf{k}}}} \left(a_{\mathbf{k}}(t) + a^{+}_{-\mathbf{k}}(t)\right) \exp(i\mathbf{kr})\, d\mathbf{r}, \end{aligned}$$

where ψ_e is the electron wave function extending in the quantum dot volume, M and n_{i} is lattice-ion mass and 3D density, V is the 3D extension of the phonon wave function, $\mathbf{e_k}$ is the displacement-sense vector, $a_{\mathbf{k}}$, $a^{+}_{-\mathbf{k}}$ are the phonon-annihilation-creation operators, and $\omega_{\mathbf{k}} = c_{\mathrm{S}}\,|\mathbf{k}|$ is the phonon frequency (c_{S} , the sound velocity).

In the following we make a 1D calculation for simplicity. Supposing that $|\psi_{\mathrm{e}}(x)|^2 = const.$ inside the quantum dot of the size L_{e}, we find

$$\int |\psi_{\mathrm{e}}(x)|^2 \exp(ikx)\, dx = \sin(kL_{\mathrm{e}}/2)\,/\,(kL_{\mathrm{e}}/2)\,.$$

When the 1D extension of the phonon wave function is L_{p}, we may make the following replacement in a 1D calculation

$$n_{\mathrm{i}}[m^{-3}]V[m^3] \rightarrow N_{\mathrm{i}}[m^{-1}]L_{\mathrm{p}}[m] \qquad (N_{\mathrm{i}} = n_{\mathrm{i}}^{1/3}),$$

and find from 1,

$$H' = i\left(\frac{\hbar C_{\mathrm{d}}^2}{2MN_{\mathrm{i}}L_{\mathrm{p}}}\right)^{1/2}\sum_k \frac{\sin\left(kL_{\mathrm{e}}/2\right)}{\left(L_{\mathrm{e}}/2\right)}\frac{1}{\sqrt{\omega_{\mathbf{k}}}}\left(a_k(t)+a_k^+(t)\right). \qquad (2)$$

We note that L_{p} may be taken to be the substrate crystal size at low temperature $T < 1\mathrm{K}$ where qubit experiments are made.

The decoherence of the electron state is studied following Ref. [7]. The initial electron state is chosen to be

$$|\psi(0)\rangle = \alpha|0\rangle + \beta|1\rangle. \qquad (3)$$

where $|0\rangle$ and $|1\rangle$ are solutions of non-interacting Hamiltonian with energy E_0 and E_1, $(E_1 > E_0)$. In the presence of an interaction t_{i}, the eigen-energy of the system is found to be given by

$$E_\pm = \frac{1}{2}\left[E_1 + E_0 \mp \sqrt{\left(E_1 - E_0\right)^2 + 4t_{\mathrm{i}}^2}\right].$$

This reveals that the upper eigen energy E_+ is higher than E_1 and the lower eigen-energy E_- is lower than E_0. Therefore, in the interaction representation, the wave function phase evolves in different sign in the two eigen states. Considering this we express the time evolution of the state function

$$\begin{aligned} |\psi(t)\rangle &= \alpha\exp\left(i\int H'dt/\hbar\right)|0\rangle + \beta\exp\left(-i\int H'dt/\hbar\right)|1\rangle \\ &\equiv A|0\rangle + B|1\rangle \qquad (4) \\ \langle\psi(t)| &= A^*\langle 0| + B^*\langle 1| \qquad (5) \\ & \qquad (6) \end{aligned}$$

The density matrix is given by

$$\rho(t) = tr\left(|\psi(t)\rangle\langle\psi(t)| \otimes w\right) \qquad (7)$$

with

$$w = \exp\left(-\frac{1}{k_{\mathrm{B}}T}\sum_k \omega_k a_k^+ a_k\right),$$

where the trace is taken over phonon quantum states. The density matrix element which related to the interference between $|1\rangle$ and $|0\rangle$ is

$$\begin{aligned} \rho(t)_{10} &= tr\left[\alpha\beta^*\exp\left(2i\int H'dt/\hbar\right)w\right] \\ &\equiv \alpha\beta^*\exp\left(-\Gamma(t)\right). \qquad (8) \end{aligned}$$

From Eqs. 2 to 8 we find

$$\Gamma(t) = \left(\frac{8C_{\mathrm{d}}^2}{\hbar M N_{\mathrm{i}} L_{\mathrm{p}} L_{\mathrm{e}}^2}\right) \int L_{\mathrm{p}} \frac{dk}{2\pi} \sin^2 \times \left(\frac{kL_{\mathrm{e}}}{2c_{\mathrm{s}}}\right) \frac{1}{\omega_k^3} (1 - \cos \omega_k t) \coth\left(\frac{\hbar\omega_k}{2k_{\mathrm{B}}T}\right)$$

$$= \left(\frac{8C_{\mathrm{d}}^2}{\pi\hbar M N_{\mathrm{i}} L_{\mathrm{e}}^2 c_{\mathrm{s}}}\right) \int d\omega_k \frac{1}{\omega_k^3} \sin^2\left(\frac{\omega_k L_{\mathrm{e}}}{2c_{\mathrm{s}}}\right) \sin^2\left(\frac{\omega_k t}{2}\right) \coth\left(\frac{\hbar\omega_k}{2k_{\mathrm{B}}T}\right)$$

We suppose $\hbar\omega_k(\mathrm{min})/k_{\mathrm{B}}T = \hbar\left(2\pi c_{\mathrm{s}}/L_{\mathrm{p}}\right)/k_{\mathrm{B}}T \ll 1$, and make the integration in the range $0 \leftrightarrow \infty$. This assumption is satisfied in ordinary experimental conditions. Considering the following ω_k dependence of the integrand factors

$$\omega_k^{-3} \to \infty \text{ when } \omega_k \to 0 \text{ and} \omega_k^{-3} \to 0 \text{ when } \omega_k \to \infty$$

gives

$$\coth \frac{\hbar\omega_k}{2k_{\mathrm{B}}T} \approx \frac{2k_{\mathrm{B}}T}{\hbar\omega_k} \gg 1$$

when $\hbar\omega_k < k_{\mathrm{B}}T$ and

$$\coth \frac{\hbar\omega_k}{2k_{\mathrm{B}}T} \approx 1$$

when $\hbar\omega_k > k_{\mathrm{B}}T$; because

$$0 \leq \sin^2\left(\omega_k L_e/2c_s\right) \sin^2\left(\omega_k t/2\right) \leq 1,$$

we neglect the contribution $\int_{k_{\mathrm{B}}T/\hbar}^{\infty} d\omega_k$ in comparison with $\int_0^{k_{\mathrm{B}}T/\hbar} d\omega_k$. Physically it corresponds to the fact that the important spectrum range of thermally excited phonon is $0 \leq \omega_k \lesssim k_{\mathrm{B}}T/\hbar$. In this approximation we get

$$\Gamma(t) = \left(\frac{8C_{\mathrm{d}}^2}{\pi\hbar M N_{\mathrm{i}} L_{\mathrm{e}}^2 c_{\mathrm{s}}}\right) \int_0^{k_{\mathrm{B}}T/\hbar} d\omega_k \frac{1}{\omega_k^3} \times \sin^2\left(\frac{\omega_k L_{\mathrm{e}}}{2c_{\mathrm{s}}}\right) \sin^2\left(\frac{\omega_k t}{2}\right) \coth\left(\frac{\hbar\omega_k}{2k_{\mathrm{B}}T}\right)$$

$$= \left(\frac{4C_{\mathrm{d}}^2 k_{\mathrm{B}}T}{\pi\hbar^2 M N_{\mathrm{i}} c_{\mathrm{s}}^3}\right) \int_0^{k_{\mathrm{B}}T/\hbar} d\omega_k \frac{\sin^2\left(\omega_k t/2\right)}{\omega_k^2},$$

where we supposed $\omega_k L_{\mathrm{e}}/2c_{\mathrm{s}} < k_{\mathrm{B}}T/\left(2\hbar c_{\mathrm{s}}/L_{\mathrm{e}}\right) < 1$, which is always satisfied in quantum dot experiment at low temperature $T < 1\mathrm{K}$. After integration, we obtain

(a) When $t << \hbar/k_{\mathrm{B}}T$

$$\Gamma(t) \approx \left(\frac{C_{\mathrm{d}}^2 k_{\mathrm{B}}^2 T^2}{\pi \hbar^3 M N_{\mathrm{i}} c_{\mathrm{s}}^3} \right) t^2 \tag{9}$$

(b) When $t >> \hbar/k_{\mathrm{B}}T$

$$\Gamma(t) = \left(\frac{C_{\mathrm{d}}^2 k_{\mathrm{B}} T}{\hbar^2 M N_{\mathrm{i}} c_{\mathrm{s}}^3} \right) t, \tag{10}$$

where we used the formula $\int_0^\infty \frac{\sin^2 ax}{x^2} dx = \frac{\pi a}{2}$. One must note that the expressions of Eqs. 9 10 are independent of the 1D wave-function extensions L_{e}, L_{p}.

In ordinary experimental conditions Eq. 10 is important in qubit operation, where the decoherence time is found from $\tau_{\mathrm{d}} = [\Gamma(t)/t]^{-1}$. In a numerical example of metal and GaAs quantum dots

$$C_{\mathrm{d}} = -6.8\mathrm{eV}$$
$$T = 10^{-3}\mathrm{K}$$
$$M = 1.4 \times 10^{-25}\mathrm{kg}$$
$$n_{\mathrm{i}} = 5 \times 10^{28}\mathrm{m}^{-3} \rightarrow N_{\mathrm{i}} = 3.6 \times 10^{9}\mathrm{m}^{-1}$$
$$c_{\mathrm{s}} = 5 \times 10^{3}\mathrm{m/s},$$

we find $\tau_{\mathrm{d}} = 0.5 \times 10^{-10}\mathrm{s}$.

Supposing the necessity of computation operation of more than 10^4 gate processing operations during τ_{d} for a quantum-dot qubit with 10 ps switching time, we achieve the appearance of device feasibility only at $T < 10^{-6}\mathrm{K}$.

2.2. JOSEPHSON JUNCTION

We study the decoherence in a SIS-type Josephson junction (or SQUID). The fundamental property of a Josephson junction is caused by the tunneling coupling of the condensed particle systems inside the two superconductor electrodes. The thermal phonon disturbance concerning the condensed state (including supercurrent) of the junction comes either from the phonon-induced fluctuation in electrode superconductivity or from that in the quantum coupling. Because of the stability of the quantum condensation with finite energy gap Δ, the former fluctuation is ignorable when $\Delta \gg k_{\mathrm{B}}T$. The latter fluctuation is also ignorable when the coupling energy E_{J} satisfies $|E_{\mathrm{J}}| \gg k_{\mathrm{B}}T$. On the other hand, the phonon-excited quasiparticles in the the electrodes make macroscopic quasiparticle current in the junction with energy dissipation. Due to the macroscopic enegy loss, the macroscopic quantum mode made by supercurrent oscillation suffers decoherence.

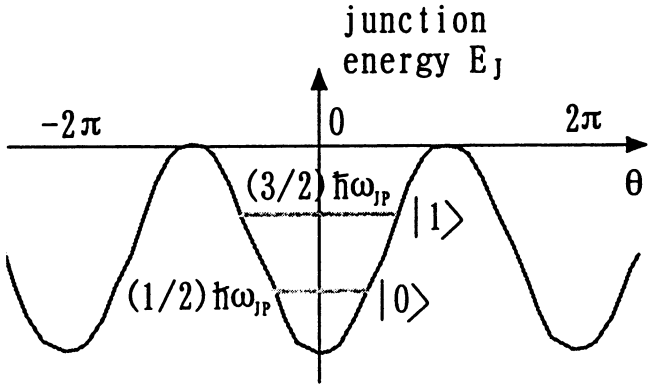

Figure 1. θ dependence of junction coupling energy and plasma oscillation levels.

The magnetic flux Φ traversed across the junction is connected with the junction phase difference θ by $\Phi = (\Phi_0/2\pi)\,\theta$, where $\Phi_0 = h/2e$ is the flux quantum. Junction current I is determined by voltage $V = d\Phi/dt$ and θ, where C and G are junction capacitance and conductance, and I_C is the critical current,

$$I = CdV/dt + GV + I_C \sin\theta = (\Phi_0 C/2\pi)\, d^2\theta/dt^2 + (\Phi_0 G/2\pi)\, d\theta/dt + I_C \sin\theta.$$

When $|\theta| < 1$ and $I = 0$,

$$d^2\theta/dt^2 + (C/G)\, d\theta/dt + (2\pi I_c/\Phi_0 C)\,\theta = 0.$$

The characteristic frequency ω satisfies

$$\omega = \left[i\,(C/2G) \pm \sqrt{(2\pi I_c/\Phi_0 C) - (C/2G)^2} \right]. \tag{11}$$

In the low loss limit $(2\pi I_C/\Phi_0 C) \gg (C/2G)^2$, we get the "Josephson plasma oscillation" frequency

$$\omega \to \omega_{JP} = \sqrt{2\pi I_C/\Phi_0 C}. \tag{12}$$

This oscillation is understood as the harmonic oscillation (see Fig. 1) in the potential well in θ space formed by the junction coupling energy

$$E_J = -(\hbar I_C/2e)\cos\theta.$$

We suppose $\Delta \gg k_B T$ and $|E_J| \simeq \hbar I_C/2e \gg k_B T$, and consider only the decoherence caused by quasiparticle current loss. We put $|0\rangle$ and $|1\rangle$ to be the ground state and the first excitation level of the oscillation mode (see Fig.1). Now the initial state ($t = 0$) is supposed to be the excited state $|1\rangle$. Since the junction has a oscillatory voltage

$$d\theta/dt = (2\pi/\Phi_0)\, d\Phi/dt = (2\pi/\Phi_0)\, V \neq 0$$

which is accompanied by a finite loss; the junction state relaxes from $|1\rangle$ to $|0\rangle$ in finite time. The decoherence time is found from Eq.11

$$\tau_{\rm d} = C/2G. \tag{13}$$

The conductance under the conditions $k_{\rm B}T \ll \Delta$, $eV < \Delta$ (Δ is the energy gap) is given by [8]

$$\begin{aligned} G &= (2G_{\rm N0}/e)\exp(-\Delta/k_{\rm B}T)\sqrt{2\Delta/(eV+2\Delta)}\,(eV+\Delta) \\ &\quad \times \sinh(eV/2k_{\rm B}T)\, I_0(eV/2k_{\rm B}T)/V \\ &\approx G_{\rm N0}(\Delta/k_{\rm B}T)\exp(-\Delta/k_{\rm B}T) \quad (V \to 0{\rm limit}), \end{aligned} \tag{14}$$

where $G_{\rm N0}$ is the normal conductance at $e|V| \gtrsim \Delta$.On the other hand, the $I_{\rm c}$ and $G_{\rm N0}$ are connected by the relationship [9]

$$I_{\rm c} = K_{\rm sc} G_{\rm N0}(\pi\Delta/2e)\tanh(\Delta/2k_{\rm B}T), \tag{15}$$

where $K_{\rm sc}$ is the strong coupling factor which takes a value $\simeq 0.8$ for Pb. Then we find from Eqs. 13~15

$$\begin{aligned} \tau_{\rm d} &= (Ck_{\rm B}T/2G_{\rm N0})\exp(\Delta/k_{\rm B}T) \\ &= (\pi K_{\rm sc} Ck_{\rm B}T/4eI_{\rm c})\exp(\Delta/k_{\rm B}T)\tanh(\Delta/2k_{\rm B}T) \end{aligned} \tag{16}$$

We study the experimental data of Ref.[10] of the measurement of the decoherence time in NbN/AlN/NbN junction, where $\tau_{\rm d} \gtrsim 10\mu s$ is obtained with the parameters $I_{\rm c} = 151\mu$A, $C = 5.8$pF, $T = 0.55$K$= 4.74\times10^{-2}$meV. The presupposed conditions are satisfied: $\Delta/k_{\rm B}T \sim 10^{1-2} \gg 1$, $|E_{\rm J}|/k_{\rm B}T \simeq \hbar I_{\rm c}/2ek_{\rm B}T = 6.5\times10^3 \gg 1$. Considering the incompleteness of NbN/AlN/-NbN junctions, we use an effective energy gap $\Delta = 1$meV instead of nominal NbN gap 2.4meV. With the parameters, (2.2.6) gives $\tau_{\rm d} \approx 17\mu s$, corresponding to the observation.

Recent development of junction fabrication technology provides us $Nb/Al_2O_3/Nb$ junction with almost ideal characteristic with parameters, for example, $\Delta = 1.3$ meV, $I_{\rm c} = 1\times10^{-4}$A ($J_{\rm c} = 10^3$A/cm^2), $R_{\rm obs}(4.2K) = 300\Omega$, $C = 0.4$pF. The estimation of decoherence time at $T = 0.5$K$= 4.31\times10^{-2}$meV based on (2.2.6) gives $\tau_{\rm d} \approx 1.4$s, and from (2.2.3) and (2.2.4) $\tau_{\rm d} \approx 25$s,. Contrary to the prevailing expectation that Josephson-junction device has short decoherence time, ideal junction may reveal long $\tau_{\rm d}$ at $T < 0.5$K comparable to nuclear spin devices.

2.3. FQHE SYSTEM

The electronic state of a FQHE system is described by the Laughlin function Ψ_{LN} [11].

$$\Psi_{LN} = \text{const.} \times \prod_{j>k} (\zeta_j - \zeta_k)^m \exp\left(-\sum_l |\zeta_l|^2 /4\right), \tag{17}$$

where $\zeta_j = (x \pm iy)/l$, $l = \sqrt{\hbar/\omega_0 M} = \sqrt{\hbar/QB}$ is magnetic length, and $\omega_0 = QB/M$ is the cyclotron frequency.

Using the transformation [12]

$$\left.\begin{array}{ll} x_+ = \left(\sum_{j=1}^N x_j\right)/\sqrt{N}, & y_+ = \left(\sum_{j=1}^N y_j\right)/\sqrt{N} \\ p_{x,+} = \left(\sum_{j=1}^N p_{jx}\right)/\sqrt{N}, & p_{y,+} = \left(\sum_{j=1}^N p_{jy}\right)/\sqrt{N} \\ x_{jk} = (x_j - x_k)/\sqrt{N}, & y_{jk} = (y_j - y_k)/\sqrt{N} \\ p_{x,jk} = (p_{jx} - p_{kx})/\sqrt{N}, & p_{y,jk} = (p_{jy} - p_{ky})/\sqrt{N} \end{array}\right\}, \tag{18}$$

Eq. 17 is rewritten

$$\Psi_{LN} = \text{const.} \times \exp\left(-|\zeta_+|^2/4\right) \times \prod_{j>k} \zeta_{jk}^m \exp\left(-|\zeta_{jk}|^2/4\right), \tag{19}$$

where we used the equality

$$\begin{aligned} \sum_{j=1}^N |\zeta_j|^2 &= \frac{1+(N-1)}{N\,l^2} \sum_{j=1}^N \left(x_j^2 + y_j^2\right) \\ &= \frac{\left(\sum_{j=1}^N x_j\right)^2 + \left(\sum_{j=1}^N y_j\right)^2}{N\,l^2} + \sum_{j>k} \frac{(x_j - x_k)^2 + (y_j - y_k)^2}{N\,l^2} \\ &= |\zeta_+|^2 + \sum_{j>k} |\zeta_{jk}|^2 . \end{aligned} \tag{20}$$

Eq. 19 is the zero-point solution of the Hamiltonian

$$H_{LN} = H_+ + \sum_{j>k} H_{jk} \tag{21}$$

$$H_+ = \left(p_{x+}^2 + p_{y+}^2\right)/2M - \omega_0 \hat{L}_{+,z}/2 + M\omega_0^2 \left(x_+^2 + y_+^2\right)/8 \qquad (L_{+,z} = 0)$$

$$H_{jk} = \left(p_{x,jk}^2 + p_{y,jk}^2\right)/2M - \omega_0 \hat{L}_{jk,z}/2 + M\omega_0^2 \left(x_{jk}^2 + y_{jk}^2\right)/8 \quad (L_{jk,z} = \hbar m)$$

$\hat{L}_z = p_x y - p_y x$, with zero-point energy and angular momentum

$$E_{LN0} = [1 + N(N-1)/2]\,\hbar\omega_0/2 \tag{22}$$

$$L_{+,z} + \sum_{j>k} L_{jk,z} = [N(N-1)/2]\,\hbar m \tag{23}$$

Using Eqs. 18˜2.3.4 and noting another equality

$$\begin{aligned}\hat{L}_{z,+} + \sum_{j>k} \hat{L}_{z,jk} \;=\; & \left\{ \frac{1}{N} \sum_j (p_{jx} y_j - p_{jy} x_j) \right. \\ & \left. + \frac{1}{N} \sum_{j>k} [(p_{jx} y_k - p_{jy} x_k) + (p_{kx} y_j - p_{ky} x_j)] \right\} \\ & + \left\{ \frac{N-1}{N} \sum_j (p_{jx} y_j - p_{jy} x_j) \right. \\ & \left. - \frac{1}{N} \sum_{j>k} [(p_{jx} y_k - p_{jy} x_k) + (p_{kx} y_j - p_{ky} x_j)] \right\} \\ = \; & \sum_{j=1}^{N} \hat{L}_{z,j}, \end{aligned} \tag{24}$$

we find that

$$H_{LN} = \sum_{j=1}^{N} \left[\left(p_{jx}^2 + p_{jy}^2 \right) / 2M + M\omega_0^2 \left(x_j^2 + y_j^2 \right) / 8 \right] - (\omega_c/2) \sum_{j=1}^{N} \hat{L}_{z,j} \tag{25}$$

Eq, 25 has a form identical to the Hamiltonian for 2D system of N non-interacting electrons in a symmetrical gauge $\mathbf{A} = (-By/2, Bx/2, 0)$. Considering Eqs. 24 25 with the constantness of the system energy and L_z in the transformation, the Laughlin solution for the Hamiltonian of expression 25 should have energy and momentum

$$E_{N0} = E_{LN0} = [1 + N(N-1)/2]\,\hbar\omega_0/2 \tag{26}$$

$$\sum_{j=1}^{N} L_{z,j} = N(N-1)\hbar m\omega_0/4 \tag{27}$$

On the other hand, the simple zero-point solution for a system of N non-interacting electrons has

$$E'_{N0} = N\hbar\omega_0/2 \tag{28}$$

$$\sum_{j=1}^{N} L_{z,j} = N\hbar m\omega_0/2. \tag{29}$$

Comparing Eqs. 26- 29 and considering the interaction between the magnetic field and magnetic moment, we find that the stability energy of Laughlin state in comparison with the non-interacting system to be

$$\Delta E = \left[\left(N^2 - 3N\right)(m-1) - 2\right]\hbar\omega_0/4.$$

Since $\Delta E > 0$ when $m \geq 2, N \geq 4$, the Laughlin state (for example at the filling factor $\nu = 1/m = 1/3$ is necessarily more stable than the many electron system of $(N \gg 1)$ non-interacting electrons.

If one dares to write an effective "1 electron Hamiltonian" H_{eff} for each electron of the Laughlin state based on 25-27, it would be

$$H_{eff} = \left(p_{jx}^2 + p_{jy}^2\right)/2M + M\omega_0^2\left(x_j^2 + y_j^2\right)/8 - (\omega_0/2)\,\hat{L}_{z,j}$$

where

$$L_{z,j} = (N-1)\hbar m\omega_0/4.$$

The zero-point solution of H_{eff} has an amplitude concentrated on the rim of a circle of radius $r \simeq \sqrt{(N-1)m}l$. When $m\tilde{}1$ and $N \gg 1$, the wave-function amplitude exists only at the periphery of 2D system. This property corresponds to the fact that quantum Hall effect appears as if all the electrons exist in the "edge state" at the periphery.[13] This may suggest that the electron-phonon interaction in the Laughlin state can be studied by considering the interaction effect of lattice vibrations on the "1 electron state" in the "edge state".

In the 1D study, the interaction Hamiltonian Eq. 1 is now

$$H' = \oint |\psi_e(s)|^2 C_{\mathrm{d}} \frac{d}{ds} u(s)\, ds = |\psi_e|^2 C_{\mathrm{d}} \oint \frac{d}{ds} u(s)\, ds,$$

where the integral path is taken around the periphery of the 2D system. If we make integration starting from a point A and ending at A, we find

$$\oint \frac{d}{ds} u(s)\, ds = u(s_A) - u(s_A) = 0.$$

This shows that the "1 electron state" in the edge has immunity from the disturbance of thermal phonon, or the decoherence time in the "1 electron state" is infinity. This long decoherence should be effective for the "1 electron state" exchanged between two Laughlin systems as "pseudo spin".

This $\tau_{\mathrm{d}} \to \infty$ property may hold when temperature is much lower than $\hbar\omega_0$ and quasiparticle excitation energy. One must note that the

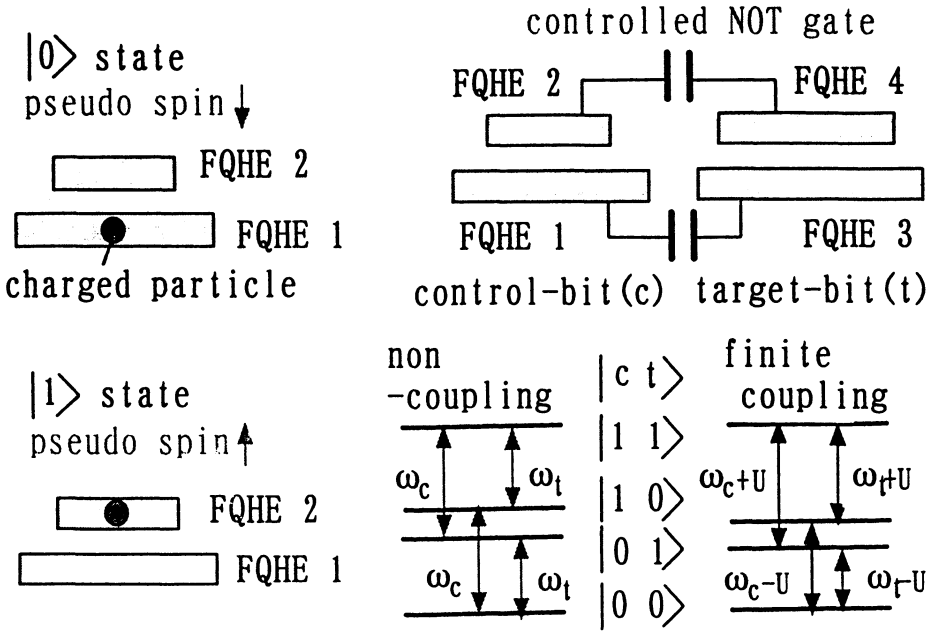

Figure 2. FQHE controlled NOT gate

quasiparticle density in FQHE is mainly determined by the filling factor ν.

3. Controlled NOT Gate Based on FQHE

The theoretical study on the controlled NOT gate using quantum dot molecule showed the adaptability to integrated circuit technology.[14] However, our study in Sec. 2.1 reveals a discouraging result with respect to the phonon-induced decoherence in the quantum dot device. If we replace the quantum dot with a small FQHE system, we may expect a controlled NOT gate having both large τ_{d} and fabrication feasibility.

It is known that the single charge exchange in two FQHE systems coupled by tunneling effect is described as a "pseudo spin".

One sees, in Fig.2 left, 2 layer FQHE systems (FQHE1 and FQHE2) connected by tunneling (overlapping integral t_{i}). Both 2D systems have areas so small that the energies E_1, E_2 ($E_2 > E_1$) in single-charge states ψ_1, ψ_2 is governed by the respective size L_1, L_2 ($L_1 > L_2$). The size de-

pendence of energy may come from electronic causes or electrostatic causes (self capacitance energy).

When $t_{\rm i} \neq 0$, the linear combination of ψ_1, ψ_2 is the eigen-state of the total system Hamiltonian with eigen-energy

$$E_{\pm} = \frac{1}{2}\left[E_2 + E_1 \mp \sqrt{(E_2 - E_1)^2 + 4t_{\rm i}^2}\right]$$

and eigen-states

$$\begin{aligned}\Psi_+ &= \frac{1}{\sqrt{2}}\left[\left(1 + \frac{E_2 - E_1}{\sqrt{(E_2 - E_1)^2 + 4t_{\rm i}^2}}\right)^{1/2} \psi_1 \right. \\ &\qquad \left. + \left(1 - \frac{E_2 - E_1}{\sqrt{(E_2 - E_1)^2 + 4t_{\rm i}^2}}\right)^{1/2} \psi_2\right] \\ \Psi_- &= \frac{1}{\sqrt{2}}\left[\left(1 - \frac{E_2 - E_1}{\sqrt{(E_2 - E_1)^2 + 4t_{\rm i}^2}}\right)^{1/2} \psi_1 \right. \\ &\qquad \left. - \left(1 + \frac{E_2 - E_1}{\sqrt{(E_2 - E_1)^2 + 4t_{\rm i}^2}}\right)^{1/2} \psi_2\right]. \end{aligned} \tag{30}$$

In the case $(E_2 - E_1)^2 \gg 4t_{\rm i}^2$, we find

$$\Psi_+ \approx \psi_1, \qquad \Psi_- \approx -\psi_2.$$

As shown in Fig.2 left, we set $|0\rangle$ state when the charge is in FQHE1 (Ψ_+, or pseudo-spin(z) $= -1/2$) and $|1\rangle$ state when it is in FQHE2 (Ψ_-, or pseudo-spin(z) $= 1/2$). By applying an appropriate voltage pulse or electro-magnetic wave as seen in Ref.[14], the 2 layer system works as a qubit.

In Fig. 2 right upside, we show the structure of FQHE controlled NOT gate. Two sets of 2 layer FQHE systems (FQHE1/FQHE2 and FQHE3/-FQHE4) are in capacitive coupling. The target bit (in abbreviation "t") FQHE3/FQHE4 has larger size than the control bit (in abbreviation "c") FQHE1/FQHE2 ($L_3, L_4 > L_2, L_1 \qquad L_3 > L_4$). In each bit, $|0\rangle$ is the state when charge is in the lower side, and $|1\rangle$ when in the upper side. In the absence of the capacitive coupling between the two bits, we write the "excitation energy" of each bit related to the charge shift

$$E_{\rm C} \approx E_2 - E_1 = \hbar\omega_{\rm C}$$

$$E_{\mathrm{t}} \approx E_4 - E_3 = \hbar\omega_{\mathrm{t}} \qquad (\omega_{\mathrm{c}} > \omega_{\mathrm{t}}).$$

The energy levels for $|\mathrm{c\ t}\rangle$ in this "non-coupling" case are shown in Fig.2 right downside. When the capacitive coupling is introduced, dipole-dipole coupling makes an interaction energy, because the pseudo-spin state in each bit corresponds to the electric dipole state. The interaction energy is $+U$ for parallel pseudo spin, and $-U$ for anti-parallel. Then the energy levels for $|\mathrm{c\ t}\rangle$ are like the "finite coupling" case shown in Fig. 2 right downside. If one applies a π pulse of frequency $\omega_t + U$, the total system makes the operation of a controlled NOT gate. A long decoherence time is expected reflecting the FQHE property.

4. Conclusion

The decoherence time with respect to the thermal phonon disturbance is theoretically studied in metal (or semiconductor) quantum dots, in superconducting junctions and in the FQHE system. For a quantum dot, the decoherence is so severe that the feasibility of the device in quantum computing is expected to appear at $T < 10^{-6}$K. Contrary to the prevailing expectation that Josephson-junction device has short decoherence time, we find that high quality junction may reveal a long τ_{d} at $T < 0.5$K comparable to τ_{d} of nuclear spin devices. On the other hand the unique property of FQHE system leads to the result that "1 electron state" suffers almost no thermal phonon disturbance so long as $k_{\mathrm{B}}T$ is much lower than the cyclotron energy and quasiparticle excitation energy. A controlled NOT gate based on the '"pseudo-spin" state is proposed, where both long decoherence time and fabrication feasibility are expected.

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